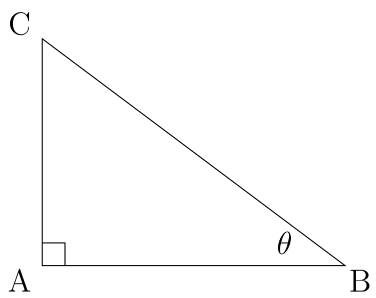
ILLAWARRA TUTORING

In this worksheet you will learn to define and use the trigonometric ratios $\sin\theta$, $\cos\theta$, and $\tan\theta$ in right-angled triangles so that you can solve problems effectively.

Easy Questions

1. Write the definitions of $\sin\theta$, $\cos\theta$, and $\tan\theta$ in a right-angled triangle. Use the terms "opposite", "adjacent", and "hypotenuse" in your definitions.

2. Look at the diagram below. For the acute angle θ, label the side opposite, the side adjacent, and the hypotenuse.



3. Using your knowledge of trigonometric ratios, calculate $\sin(30°)$.

4. In a right-angled triangle, if the length of the adjacent side for angle θ is 4 and the hypotenuse is 5, calculate $\cos\theta$.

5. State which trigonometric ratio relates the length of the side opposite to the length of the side adjacent in a right-angled triangle.

Intermediate Questions

6. In a right-angled triangle the length of the side opposite an angle θ is 7. Write an expression for the hypotenuse in terms of $\sin\theta$.

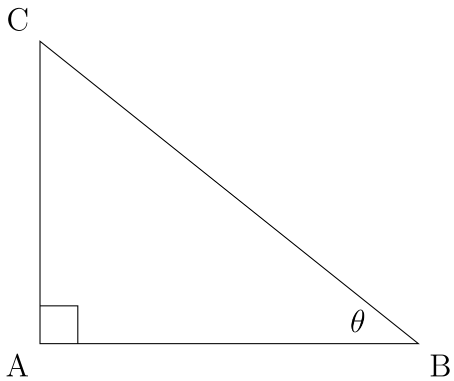
7. If $\cos\theta$ is defined by $\cos\theta = \frac{a}{10}$, express a in terms of $\cos\theta$.

8. Given $\tan\theta = \frac{x}{12}$, rearrange the equation to express x in terms of $\tan\theta$.

9. Given that in a right-angled triangle $\sin\theta = \frac{3}{5}$, use the identity $\sin^2\theta + \cos^2\theta = 1$ to write an expression for $\cos\theta$ in terms of $\sin\theta$.

10. Using your answer from the previous question, calculate $\cos\theta$ when $\sin\theta = \frac{3}{5}$ (take the positive square root as θ is acute).

11. Draw the right-angled triangle provided below and label all sides and the acute angle θ. Then write the definitions of $\sin\theta$, $\cos\theta$, and $\tan\theta$ for your diagram.



12. Refer to the diagram in Question 11. If the side opposite to angle θ has length 4 and the adjacent side has length 3, calculate $\tan\theta$.

13. On a blank sheet of paper, draw a right-angled triangle and label one of its acute angles as θ. Then, write down the formulas for $\sin\theta$, $\cos\theta$, and $\tan\theta$ using your diagram.

14. Solve for x if $\sin\theta = \frac{x}{20}$ and it is given that $\sin\theta = \frac{1}{2}$.

15. If $\cos\theta = 0.6$, use the Pythagorean identity to determine $\sin\theta$ for an acute angle θ.

16. In a right-angled triangle, if $\tan\theta = 0.75$ and the length of the adjacent side is 8, calculate the length of the side opposite to θ.

17. Show algebraically that $\tan\theta$ can be expressed as $\tan\theta = \frac{\sin\theta}{\cos\theta}$.

18. True or False: For a given acute angle in any right-angled triangle, the trigonometric ratios $\sin\theta$, $\cos\theta$, and $\tan\theta$ remain constant. Provide a brief explanation for your answer.

19. In a right-angled triangle, the sine of an acute angle θ is given by $\sin\theta = \frac{5}{h}$, where h represents the hypotenuse. Find the value of h if $\sin\theta = 0.5$.

20. If $\tan\theta = k$ for an acute angle θ, derive expressions for $\sin\theta$ and $\cos\theta$ in terms of k.

Hard Questions

21. Using the definitions of $\sin\theta$ and $\cos\theta$ in a right-angled triangle, prove that $\sin^2\theta + \cos^2\theta = 1$.

22. Prove that if $\tan\theta = k$ then $\sin\theta = \dfrac{k}{\sqrt{1+k^2}}$ and $\cos\theta = \dfrac{1}{\sqrt{1+k^2}}$, for an acute angle θ.

23. In a right-angled triangle with sides of lengths a, b, and hypotenuse c, let θ be the acute angle opposite side a. Write $\sin\theta$ in terms of a and c. Then, using the relationship $a^2 + b^2 = c^2$, discuss briefly why $\sin\theta \leq 1$.

24. Suppose that in a right-angled triangle $\sin\theta = \dfrac{2x}{5}$ and $\cos\theta = \dfrac{\sqrt{25-4x^2}}{5}$ for $0 < x \leq \dfrac{5}{2}$ and $0 < \theta < 90^\circ$. Show that $\tan\theta = \dfrac{2x}{\sqrt{25-4x^2}}$ and state the constraints on x for the expression to be valid.

25. In a right-angled triangle with side lengths in the ratio $3 : 4 : 5$, calculate $\sin\theta$, $\cos\theta$, and $\tan\theta$ for the acute angle opposite the side of length 3.

26. Using pen and paper, construct a right-angled triangle with an acute angle of your choice. Measure the lengths of the sides and verify that the definitions of $\sin\theta$, $\cos\theta$, and $\tan\theta$ hold true. Write a short explanation of your findings.

27. For an acute angle θ in a right-angled triangle, if $\sin\theta = \dfrac{\sqrt{2}}{2}$, determine $\cos\theta$ and $\tan\theta$ without using a calculator.

28. A right-angled triangle has side lengths 5, 12, and 13. Identify the smallest acute angle and compute $\sin\theta$, $\cos\theta$, and $\tan\theta$ for that angle.

29. Given that $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$, solve algebraically for $\cos\theta$ in terms of $\tan\theta$ and $\sin\theta$.

30. For an acute angle θ in a right-angled triangle, if $\sin\theta = 0.8$, derive and simplify the expressions for $\cos\theta$ and $\tan\theta$ in radical form.