This worksheet focuses on the sine rule: learning how to use the relation $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ to solve non–right-angled triangles effectively. Work through the questions in order.

Easy Questions

1. Write down the sine rule for any triangle with sides a, b, c and opposite angles A, B, C.

2. In any triangle the sum of the interior angles is $180°$. Given that $A = 40°$ and $B = 70°$, write an expression for the third angle C.

3. In a triangle, if $A = 30°$, $B = 45°$ and the side opposite A is $a = 10$, use the sine rule to set up an expression to find side b. Do not solve it completely.

4. Consider a triangle where $A = 50°$, $B = 60°$, and the side opposite A is $a = 12$. Write an expression using the sine rule to find side b. Simplify your expression as much as possible.

5. Below is a diagram of a triangle. Using the diagram, label the sides and angles. Given that $B = 40°$, $C = 70°$ and the side opposite B is $b = 9$, write the sine rule expression to find side c.
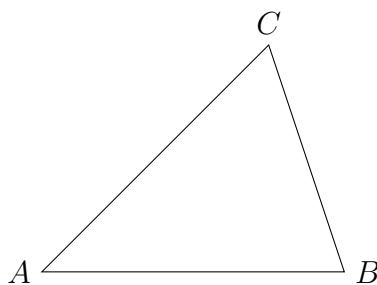
Intermediate Questions

6. In a triangle, given $A = 40°$, $B = 60°$, and side $a = 10$, first find C, then use the sine rule to find side b.

7. Given that $A = 30°$, $B = 100°$ and side $c = 15$ in a triangle, find the third angle and then set up the sine rule to solve for side a.

8. In a triangle, let $A = 50°$, $B = 80°$ and side $a = 7$. Find angle C and use the sine rule to determine side c.

9. For a triangle with $A = 65°$, $C = 55°$, and side $c = 9$, compute angle B and then use the sine rule to find side a.

10. A triangle has $A = 25°$, $B = 80°$, and side $a = 8$. Compute angle C, then use the sine rule to determine side b.

11. In a triangle, if $B = 70°$, $C = 40°$, and side $c = 10$, first determine angle A and then use the sine rule to find side b.

12. For a triangle with $A = 55°$, $B = 65°$, and side $a = 11$, find the third angle C and use the sine rule to solve for side b.

13. Consider the triangle shown below. Given that $A = 35°$, $B = 75°$, and side $b = 14$, first determine angle C and then set up the sine rule to calculate side a.
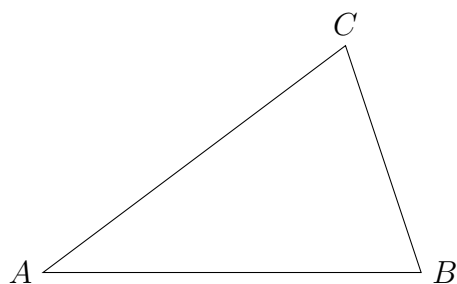
14. In a triangle, if $C = 60°$, $A = 45°$, and side $c = 13$, first find the remaining angle B, then use the sine rule to find side a.

15. A triangle has $A = 50°$, side $a = 15$, and $B = 75°$. Determine angle C and then use the sine rule to calculate side b.

16. In a triangle with $A = 50°$, $C = 40°$, and side $a = 10$, first compute angle B and then apply the sine rule to find side c.

17. Consider a triangle with sides $a = 8$, $b = 10$, and angle $A = 30°$. Use the sine rule to find the possible value(s) for angle B. Explain if an ambiguous case exists.

18. Given a triangle where $a = 9$, $A = 40°$, and $B = 55°$, determine angle C and then use the sine rule to solve for side b. Round your answer to one decimal place.

19. In a triangle with side $a = 20$, $A = 100°$, and $B = 30°$, first obtain angle C and then solve for side b using the sine rule.

20. For a triangle where $a = 12$, $A = 70°$, and $B = 40°$, find angle C and use the sine rule to determine side b.

Hard Questions

21. A triangle has side $a = 8$, angle $A = 45°$, and side $b = 10$. Determine all possible triangles that satisfy these conditions using the sine rule.

22. In a triangle, given $a = 7$, $b = 10$ and $A = 30°$, use the sine rule to determine the possible values for angle B. Clearly explain the ambiguity.

23. A triangle has side $a = 9$, side $b = 12$, and angle $A = 40°$. Determine both possible measures of angle B (if they exist) and then use the sine rule to find side c.

24. Consider the circumstance in which the sine rule yields two possible values for an angle. Prove that only one of these possible angles will yield a valid triangle by discussing the sum of the angles.

25. Derive an expression for side c in terms of side a, angle A and angle C using the sine rule.

26. In triangle ABC, given that $a = 15$, $A = 80°$, and $B = 50°$, compute the third angle C and then calculate side b using the sine rule. Give your answer for side b rounded to three decimal places.

27. A triangle has angles $A = 30°$, $B = 45°$, and side $c = 14$. Determine the remaining angle and use the sine rule to calculate side a. Round your answer to one decimal place.

28. In a triangle, if side $a = 10$, side $b = 12$, and angle $A = 40°$, use the sine rule to determine the possible values for angle B. Explain your reasoning.

29. In triangle ABC, the side opposite angle A is 16, angle $A = 55°$, and angle $B = 65°$. Determine angle C and then compute side b using the sine rule.

30. A triangle has side $a = 22$, angle $A = 85°$, and angle $B = 50°$. Calculate angle C and use the sine rule to find side b. Provide your answers with appropriate rounding.