In this worksheet you will learn to apply the cosine rule to find unknown sides or angles in triangles where the sine rule is not applicable. You will practise substituting known values into the cosine rule and rearranging the formula to solve for the desired quantity.

Easy Questions

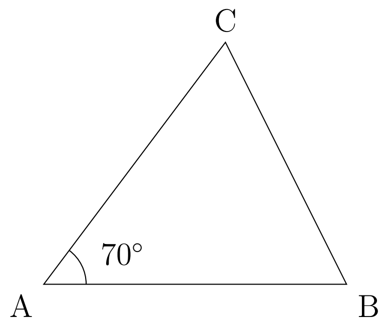
1. In triangle ABC, let $a = 5$, $b = 7$, and the included angle $C = 60°$. Use the cosine rule
$$c^2 = a^2 + b^2 - 2ab\cos C$$
to calculate c.

2. In triangle ABC, let $b = 8$, $c = 10$, and the included angle $A = 45°$. Use the cosine rule
$$a^2 = b^2 + c^2 - 2bc\cos A$$
to find the length of side a.

3. In triangle ABC, the side lengths are $a = 6$, $b = 8$, and $c = 10$. Use the cosine rule in the form
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$
to find the measure of angle C (the angle opposite side c).

4. In triangle ABC, the sides have lengths $a = 4$, $b = 5$, and $c = 6$. Use the cosine rule
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$
to calculate the measure of angle C.

5. Refer to the diagram below. In triangle ABC, $AB = 9$, $AC = 7$, and the measure of angle $A = 70°$. Use the cosine rule
$$BC^2 = AB^2 + AC^2 - 2\,(AB)(AC)\cos A$$
to determine the length of side BC.



Intermediate Questions

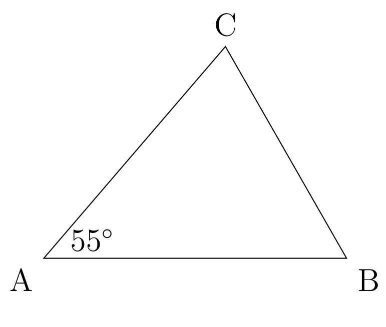
6. In triangle PQR, given $PQ = 8$, $PR = 15$, and the included angle $QPR = 30°$, use the cosine rule
$$QR^2 = PQ^2 + PR^2 - 2\,(PQ)(PR)\cos QPR$$
to calculate the length of side QR.

7. In triangle ABC, let $a = 10$, $b = 14$, and the included angle $C = 120°$. Use the cosine rule
$$c^2 = a^2 + b^2 - 2ab\cos C$$
to find the length of side c.

8. In triangle ABC, the side lengths are $a = 7$, $b = 9$, and $c = 12$. Use the cosine rule in the form
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$
to determine the measure of angle C (opposite side c).

9. In triangle PQR, let $PQ = 8$, $PR = 6$, and the included angle $QPR = 45°$. Apply the cosine rule to find side QR.

10. Refer to the diagram below. In triangle ABC, $AB = 12$, $AC = 9$, and angle $A = 55°$. Use the cosine rule
$$BC^2 = AB^2 + AC^2 - 2\,(AB)(AC)\cos A$$
to calculate the length of side BC.



11. In triangle ABC, let $BC = 10$, $AC = 14$, and the included angle $B = 40°$. Use the cosine rule to determine the length of side AB.

12. In triangle ABC, the sides are $a = 9$, $b = 11$, and $c = 13$. Use the cosine rule

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

to calculate the measure of angle A (opposite side a).

13. A triangular park has two sides measuring 20 m and 30 m with an included angle of 80°. Use the cosine rule to find the length of the third side.

14. In triangle ABC, the sides measure $a = 9$, $b = 10$, and $c = 7$. Apply the cosine rule in the angle form to compute the measure of the angle opposite the side with length 7.

15. In a triangle with sides a, b, and c and with included angle C between a and b, write an expression for c using the cosine rule. Simplify your answer.

16. In triangle XYZ, let $x = 12$, $y = 9$, and the included angle $Z = 30^\circ$ (between sides x and y). Use the cosine rule

$$z^2 = x^2 + y^2 - 2xy \cos Z$$

to find the length of side z.

17. In triangle ABC, suppose side $a = 5$, side $b = 8$, and the angle opposite side a is 60°. First, explain why the cosine rule should be used in this case and then find side a (verify consistency).

18. In an equilateral triangle all sides are equal. Assume $a = b = c = s$. Use the cosine rule

$$c^2 = a^2 + b^2 - 2ab \cos C$$

to show that $\cos C = \frac{1}{2}$ and hence explain why each angle is 60°.

19. Given a triangle with sides a, b, and c, rearrange the cosine rule to express $\cos C$. Then, for a triangle where $a = 8$, $b = 9$, and $c = 7$, compute $\cos C$.

20. In triangle ABC, let $a = 15$, $b = 22$, and $c = 18$. Use the cosine rule in the form

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

to calculate the measure of angle C.

Hard Questions

21. In triangle ABC, let $a = 13$, $b = 17$, and the included angle $C = 95^\circ$. Use the cosine rule

$$c^2 = a^2 + b^2 - 2ab \cos C$$

to determine the length of side c.

22. In triangle ABC, suppose $a = x$, $b = 2x$, and the included angle $C = 60^\circ$. Use the cosine rule to express the length of side c in terms of x. Simplify your answer.

23. A surveyor measures two distances from point A to points B and C as $AB = 50\,\text{m}$ and $AC = 45\,\text{m}$, with the included angle $A = 110^\circ$. Use the cosine rule to determine the distance BC between points B and C.

24. Using vector notation and the dot product, provide a short proof of the cosine rule. (Hint: Express the vector representing one side in terms of the others.)

25. In triangle PQR, let $PQ = 21$, $PR = 28$, and the included angle $QPR = 75^\circ$. Use the cosine rule to calculate the length of side QR.

26. In triangle ABC, given that $AB = 15$, $AC = 20$, and the included angle $A = 35^\circ$, apply the cosine rule

$$BC^2 = AB^2 + AC^2 - 2\,(AB)(AC)\cos A$$

to determine BC.

27. In triangle ABC, the sides measure $a = 10.5$, $b = 13.2$, and $c = 7.8$. Use the cosine rule in the form

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

to compute the measure of angle A (the angle opposite side a).

28. Rearrange the cosine rule to solve for $\cos A$. Then, for a triangle where $a = 9$, $b = 11$, and $c = 15$, use your rearranged formula to calculate the measure of angle A.

29. In an equilateral triangle with side length s, show that substituting $a = b = c = s$ into the cosine rule yields

$$\cos A = \frac{s^2 + s^2 - s^2}{2s^2} = \frac{1}{2}$$

and conclude that each angle is 60°.

30. A sailing boat is at point P. Two landmarks, T1 and T2, are observed such that $PT1 = 30\,\text{m}$ and $PT2 = 40\,\text{m}$ with an angle of 100° between the lines of sight. Use the cosine rule

$$T1T2^2 = PT1^2 + PT2^2 - 2\,(PT1)(PT2)\cos 100^\circ$$

to determine the distance between the two landmarks.