ILLAWARRA TUTORING

This worksheet contains 30 questions in Mixed problems that challenge you to combine different trigonometric principles learned so far. You will use trigonometric ratios, the sine rule, the cosine rule, and the area formula of a triangle to solve a variety of problems. Read each instruction carefully and answer the questions in full.

Easy Questions

1. In a right triangle ABC with $\angle A = 40^\circ$ and hypotenuse 12, calculate the length of the side opposite $\angle A$.

2. In triangle DEF, the angles are $\angle D = 50^\circ$, $\angle E = 60^\circ$, and the side opposite $\angle D$ measures 10. Find the length of the side opposite $\angle E$ using the sine rule.

3. In triangle PQR, with $PQ = 7$, $PR = 10$, and $\angle P = 45^\circ$, use the cosine rule to determine the length of side QR.

4. In a right triangle XYZ, the side opposite $\angle X = 35^\circ$ is 8. Calculate the hypotenuse of the triangle.

5. In triangle LMN, the angles are 40°, 60°, and 80°. If the side opposite the 40° angle measures 5, determine the length of the side opposite the 60° angle using the sine rule.

Intermediate Questions

6. In right triangle ABC with $\angle B = 40^\circ$, if side $AC = 6$ (the leg opposite $\angle B$), find the length of side BC (the adjacent leg) and hence determine the area of the triangle.

7. In triangle DEF, $DE = 8$, $DF = 10$, and $\angle D = 45^\circ$. First, use the cosine rule to find side EF. Then, apply the sine rule to determine one of the remaining angles.

8. In triangle GHI, $\angle G = 55^\circ$, $\angle H = 65^\circ$, and the side opposite $\angle G$ is 9. Use the sine rule to find the length of the side opposite $\angle H$.

9. Write down your answer: Two observers stand at points A and B on level ground, 50 metres apart. From A, the angle of elevation to a hot-air balloon is 40°, and from B it is 35°. Using an appropriate diagram, determine the height of the balloon.

10. Two ships, P and Q, are observed from a harbour. The bearing of ship P is 30° and that of ship Q is 70°. If the distance between the ships is 4 km, determine the distance from the harbour to ship P using trigonometric principles.

11. In triangle RST, $RS = 9$, $ST = 7$, and the altitude from vertex T onto side RS is 4. Determine $\angle S$ by first finding the area of the triangle and then applying the sine area formula.

12. In triangle JKL, $JK = 12$, $JL = 9$, and $\angle J = 70°$. First, calculate the area of the triangle using the formula $\frac{1}{2}ab\sin(C)$, then use the cosine rule to determine side KL.

13. A triangular park has one side measuring 20 metres with the adjacent angles $50°$ and $60°$. Use the sine rule to find the lengths of the other two sides and calculate the perimeter of the park.

14. In triangle MNO, where $MN = 14$, $\angle M = 50°$, and $\angle N = 60°$, use the sine rule to determine the length of side NO.

15. In triangle STU, $ST = 10$, $TU = 7$, and $\angle T = 40°$. First, apply the cosine rule to find side SU, and then use the sine rule to find one of the remaining angles.

16. In triangle VWX, $\angle V = 30°$, side $v = 8$, and side $w = 10$. Investigate the possible measures for $\angle W$ using the sine rule and determine if one or two solutions exist.

17. In triangle YZA, with $\angle Y = 45°$, $\angle Z = 70°$, and the side opposite $\angle Y$ is 11, use the sine rule to determine the missing side and then compute the area of the triangle.

18. Two triangles share a common side. In triangle ABC, $AB = 8$, $\angle A = 50°$, and $\angle B = 60°$. In an adjacent triangle ABD sharing side AB, $\angle BAD = 40°$ and side $AD = 7$. Use trigonometric principles to find side BD.

19. A rectangular field has a triangular extension on one side. The triangle has a base of 15 metres (which is also a side of the rectangle) and an angle of $30°$ at the base. If the altitude of the triangle from the vertex opposite the base is 4 metres, find the lengths of the other sides of the triangle using appropriate trigonometric methods.

20. In triangle BCD, $BC = 9$, $CD = 11$, and $\angle B = 55°$. First use the cosine rule to determine side BD, then apply the sine rule to find $\angle D$.

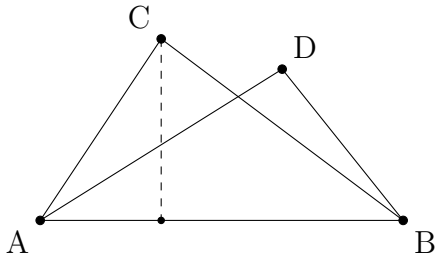
Hard Questions

21. In triangle ABC, let $a = 9$, $b = 7$, and $\angle A = 60°$. First, use the cosine rule to determine side c, then apply the sine rule to find $\angle C$. Show all your working.

22. In triangle DEF, with $\angle D = 40°$, $d = 8$, and $e = 12$, use the sine rule to determine the possible values for $\angle E$. State clearly whether one or two solutions exist and justify your reasoning.

23. In triangle GHI, given $\angle G = 80°$, $\angle H = 45°$, and side $g = 10$, use the sine rule to find side h and then compute the area of the triangle using the formula $\frac{1}{2}ab\sin(C)$.

24. In the figure below, two non-right triangles share the common base AB. In triangle ABC, $\angle CAB = 50°$ and side $AC = 9$. In triangle ABD, $\angle BAD = 40°$ and side $AD = 7$. Using the diagram, determine the height from point C to the line AB.



25. A navigator on a ship observes a lighthouse from two different points. From point P, the bearing of the lighthouse is 20°, and from point Q, 30 km due east of P, the bearing is 10°. Using an appropriate diagram, determine the distance from point P to the lighthouse.

26. From a point on level ground, the top of a tower is observed at an angle of elevation of 35°. Moving 50 metres further from the tower, the angle of elevation is measured as 30°. Combining trigonometric ratios and the sine rule where necessary, determine the height of the tower.

27. In triangle JKL, with side $j = 11$, side $k = 13$, and $\angle J = 65°$, first use the cosine rule to find side l, and then apply the sine rule to determine $\angle K$. Detail all steps.

28. In triangle MNO, $MN = 10$, $MO = 6$, and the area is given as 15. Use the area formula Area $= \frac{1}{2} MN\, MO \sin(\angle M)$ to find $\angle M$, and then determine the remaining angles of the triangle.

29. In triangle PQR, $\angle P = 50°$, side $p = 9$, and side $q = 11$. First, use the sine rule to find $\angle Q$, then apply the cosine rule to calculate side r, and finally compute the area of the triangle.

30. In triangle STU, side $ST = 8$, $\angle S = 55°$, and the altitude from vertex U to side ST is 4. Using appropriate trigonometric methods and the area formula, determine the length of side TU. (Hint: Express the area in two different ways and equate them to solve for the unknown side.)