ILLAWARRA TUTORING

In this worksheet you will challenge yourself with complex problems that require you to combine different trigonometric principles such as basic trigonometric ratios in right-angled triangles, applications involving the sine rule, cosine rule and area calculations. Read each question carefully and show all working.

Easy Questions

1. Use the sine ratio to find the length of the side opposite the given angle in a right-angled triangle. In a right-angled triangle, if one acute angle is 30° and the hypotenuse is 10, compute the length of the side opposite this angle.

2. Use the cosine ratio to determine the hypotenuse. In a right-angled triangle, if one acute angle is 60° and the side adjacent to this angle is 4, find the length of the hypotenuse.

3. Use the tangent ratio to find the missing side. In a right-angled triangle, if one acute angle is 45° and the side adjacent to this angle is 5, calculate the length of the side opposite this angle.

4. Use trigonometric ratios to find missing sides. In a right-angled triangle, if one acute angle is 35° and the side adjacent to it is 6, determine the hypotenuse and the side opposite the 35° angle.

5. Use an inverse trigonometric function to find the measure of an angle. In a right-angled triangle, if the side opposite an acute angle is 3 and the side adjacent is 3, determine the measure of that angle.

Intermediate Questions

6. Use the sine rule. In triangle ABC, let $\angle A = 45^\circ$, $\angle B = 60^\circ$, and side $a = 8$. Find the length of side b.

7. Use the cosine rule. In triangle ABC, where $a = 7$, $b = 9$, and the included angle $\angle C = 60^\circ$, calculate the length of side c.

8. Solve a ladder problem. A ladder of length 10 leans against a vertical wall with its base 3 from the wall. Determine (a) the height at which the ladder meets the wall, and (b) the angle between the ladder and the ground.

9. In a right-angled triangle the side opposite a 30° angle measures 4. Compute (a) the hypotenuse and (b) the side adjacent to the 30° angle.

10. A surveyor measures the angle of elevation to the top of a tower from a point and then walks 10 metres closer, where the angle of elevation becomes $70°$. If the initial angle of elevation was $50°$, determine the height of the tower.

11. Find the area and an altitude. In a triangle, two sides have lengths 9 and 12, with an included angle of $60°$. (a) Calculate the area of the triangle using the sine formula. (b) Determine the length of the altitude to the side of length 12.

12. Use the cosine rule. In a triangle, two sides measure 8 and 10, and the included angle is $30°$. Find the length of the third side.

13. Apply the sine rule. In an oblique triangle, sides a and b measure 7 and 9, with angles opposite these sides measuring $40°$ and $65°$ respectively. Find the length of side c.

14. In a right-angled triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into segments of length 4 and 9. Calculate the length of the altitude.

15. In a right-angled triangle the two acute angles differ by $20°$ and the hypotenuse is 13. Determine the measures of the acute angles and calculate the lengths of the sides opposite these angles.

16. Use two angles of elevation in a surveying problem. An observer measures the angle of elevation to the top of a tower as $30°$ from point A. After moving 5 metres closer to the tower to point B, the angle of elevation is $45°$. Find the height of the tower.

17. In a triangle, two sides measure 5 and 7, and the area is 6. Determine the measure of the included angle between these two sides.

18. In a triangle, two sides measure 6 and 8 with an included angle of $50°$. (a) Compute the area using the sine formula. (b) Then, find the length of the third side using the cosine rule, and (c) verify the area using Heron's formula.

19. A triangular plot has two sides of lengths 30 m and 40 m with an included angle of $60°$. (a) Find the area of the plot. (b) Determine the measures of the other two angles.

20. Two points on a level horizontal line are 50 m apart. From the nearer point the angle of elevation to the top of a hill is $32°$, and from the farther point it is $28°$. Determine the height of the hill.

Hard Questions

21. In triangle ABC, side $AB = 13$, side $AC = 10$, and $\angle A = 40°$. (a) Compute the length of side BC using the cosine rule. (b) Then, determine the measures of $\angle B$ and $\angle C$ using the sine rule.

22. A ladder leans against a vertical building. When positioned normally it makes an angle of $75°$ with the ground. When the bottom is moved 2 m further from the wall the angle reduces to $60°$. Determine (a) the length of the ladder and (b) the height at which it touches the building.

23. In triangle ABC the measure of $\angle B$ is twice that of $\angle C$. If side $a = 10$ and side $b = 14$, use trigonometric principles to determine the measure of $\angle A$.

24. In triangle ABC the sides are in arithmetic progression and the largest angle is 120°. Determine the lengths of the sides in terms of a suitable parameter.

25. From two points 30 m apart along a straight line towards a tower, the angles of elevation to the top of the tower are measured as 15° and 20°. Determine the height of the tower.

26. In triangle ABC, side $AB = 8$, side $AC = 6$, and the median from A (to side BC) has length 5. Combine trigonometric principles with the median formula to determine $\angle A$.

27. In triangle ABC, $\angle A = 50^\circ$, side $AC = 9$, and the altitude from A to side BC divides BC into segments of lengths 3 and x. Determine the value of x.

28. In an isosceles triangle ABC with $AB = AC$, if side $BC = 10$ and the altitude from A to BC is 6, determine the lengths of sides AB and AC.

29. Two observers at points P and Q, which are 40 m apart, measure the angles of elevation to a hilltop as 32° and 28°, respectively. Assuming the points lie on a straight line towards the hill, determine (a) the distance from the nearer observer to the hill and (b) the height of the hill.

30. A quadrilateral $ABCD$ is divided by the diagonal AC. In triangle ABC, side $AB = 8$, side $BC = 6$, and $\angle ABC = 45^\circ$. In triangle ADC, sides $AD = 7$ and $CD = 5$. (a) Find the length of AC by applying the cosine rule in each triangle and (b) verify that the results are consistent.