



In this worksheet you will master the techniques to calculate the area of a triangle using trigonometric methods. You will use the formula

$$A = \frac{1}{2}ab \sin C$$

where a and b are two sides and C is the included angle.

Easy Questions

1. Calculate the area of a triangle with side lengths 8 and 10 units and an included angle of 30° .
2. Find the area of a triangle with sides 5 cm and 12 cm forming a right angle.
3. Explain why the area of a triangle can be written as $A = \frac{1}{2}ab \sin C$. In your answer, identify the role of the height when a side is chosen as the base.
4. Calculate the area of a triangle with sides 7 units and 9 units and an included angle of 120° .
5. A triangle has an area of 20 cm^2 , one side measuring 5 cm and an included angle of 60° . Calculate the value of the product $b \sin 60^\circ$ where b is the other side.

Intermediate Questions

6. A triangle has sides 12 cm and 15 cm with an included angle of 45° . Determine its area.
7. In triangle ABC, side $AB = 9$ cm, side $AC = 11$ cm and the included angle at A is 30° . Compute the area of triangle ABC.
8. Given a triangle with sides $a = 10$ cm and $b = 14$ cm and an included angle of 60° , find its area.
9. A triangle has an area of 36 cm^2 and two sides of lengths 8 cm and b cm, with an included angle of 45° . Find b .
10. In triangle PQR, side $PQ = 13$ cm, side $PR = 10$ cm and the included angle $QPR = 120^\circ$. Find its area.

11. Derive the formula $A = \frac{1}{2}ab \sin C$ by considering a triangle with a chosen base and its corresponding height.
12. A triangle has an area of 50 cm^2 , one side of 10 cm and an included angle of 30° . Find the value of $b \sin 30^\circ$ where b is the other side.
13. Calculate the area of a triangle with sides 16 cm and 20 cm , and an included angle of 15° .
14. In triangle DEF, with side lengths $DE = 7 \text{ cm}$ and $DF = 9 \text{ cm}$ and an included angle of 110° , compute its area.
15. A triangle with sides 8 cm and 16 cm has an area $A = \frac{1}{2} \times 8 \times 16 \times \sin \theta$. For what value of θ is A maximised?
16. Two triangles share an included angle and a common side. If the other side in the first triangle is twice as long as the corresponding side in the second triangle, explain how their areas compare.
17. A triangle with sides 7 cm and 10 cm has an area of 17.5 cm^2 . Find $\sin(\text{included angle})$ and hence the measure of this angle.
18. Use the formula $A = \frac{1}{2}ab \sin C$ to explain how two triangles with different side lengths may have the same area.
19. A triangle has an area of 24 cm^2 . If one side is 4 cm and the included angle is 90° , find the length of the other side.
20. In triangle GHI, side $GH = 11 \text{ cm}$, side $HI = 13 \text{ cm}$, and the included angle is 80° . Find its area.

Hard Questions

21. A triangle has sides of 9 cm and 15 cm , and its area is 27 cm^2 . Determine the measure of the included angle.
22. A triangle has sides 12 cm and 18 cm with an included angle that is not acute. If its area is 54 cm^2 , determine the measure of the angle.
23. In triangle JKL, side $JK = 11 \text{ cm}$, side $JL = 10 \text{ cm}$, and the area is 22 cm^2 . Calculate $\sin J$.
24. The area of triangle MNO is given by $A = \frac{1}{2} \times 14 \times x \times \sin 40^\circ$. If $A = 35 \text{ cm}^2$, solve for x in terms of $\sin 40^\circ$.
25. A triangle has one side of length 4 cm that is fixed and an area of 6 cm^2 when paired with another side $b \text{ cm}$ and an included angle C . Express b in terms of $\sin C$.
26. In triangle XYZ, sides $XY = 8 \text{ cm}$ and $XZ = 15 \text{ cm}$ yield an area of 30 cm^2 . Determine $\sin(\text{included angle})$.

27. A triangle is such that its area is given by $A = 4 \sin \theta$ and one of its sides is fixed at 8 cm. Express the unknown side b in terms of θ using the area formula.
28. In triangle ABC, if sides of lengths 10 cm and 12 cm contain an angle C such that the area is 30 cm^2 , determine $\sin C$.
29. Prove that for a fixed included angle, increasing one of the sides of a triangle increases the area proportionally. Give a brief explanation using the area formula.
30. A scalene triangle with sides of lengths a and b has maximum area for given a and b when the included angle is a right angle. Explain why, and then calculate the maximum area when $a = 9 \text{ cm}$ and $b = 12 \text{ cm}$.