ILLAWARRA TUTORING

In this worksheet you will master techniques to calculate the area of a triangle using trigonometric methods. In particular, you will use the formula Area $= \frac{1}{2}ab\sin C$ where a and b are two sides of a triangle and C is the included angle (in degrees).

Easy Questions

1. Calculate the area of a triangle with sides $a = 8$, $b = 5$ and included angle $C = 30°$. Use the formula $\frac{1}{2}ab\sin C$.
2. Find the area of a triangle with sides $a = 7$, $b = 10$ and included angle $C = 60°$.
3. Determine the area of a triangle where $a = 4$, $b = 9$ and $C = 45°$.
4. Explain what happens to the area of a triangle if the included angle $C = 0°$. Justify your answer using the area formula.
5. Calculate the area of a triangle with sides $a = 6$, $b = 8$ and $C = 90°$, given that $\sin 90° = 1$.

Intermediate Questions

6. Compute the area of a triangle with $a = 12$, $b = 9$ and $C = 75°$.
7. A triangle has sides $a = 10$, $b = 6$ and an area of 15. Find the measure of the included angle C. (Hint: Rearrange the formula to solve for $\sin C$.)
8. In a triangle with $a = 8$ and $b = 5$, the area is 10. Show that $\sin C = \dfrac{2 \times 10}{8 \times 5}$ and simplify your answer.
9. A triangle with sides $a = 6$ and $b = 8$ has an area of 24. Find $\sin C$ and hence determine C.
10. If a triangle has sides a and b with included angle C, express the area A in terms of a, b, and $\sin C$.
11. Calculate the area of a triangle with $a = 5$, $b = 12$, and an included angle C such that $\sin C = 0.5$.
12. Determine the area of a triangle with sides $a = 14$, $b = 9$, and included angle $C = 120°$.

13. A triangle with sides $a = 9$ and $b = 10$ has an area of 20. Find $\sin C$, and then calculate the measure of C.

14. In a triangle with $a = 5$, $b = 6$, the area equals 15. Determine the measure of the included angle C.

15. If a triangle has two equal sides each of length 10 and the included angle is $60°$, compute its area.

16. A triangle has sides a and b, and an included angle C. Explain how changing C affects the area, keeping a and b constant.

17. Given that $\frac{1}{2}ab\sin C = 18$, with $a = 6$ and $b = 8$, calculate $\sin C$ and then deduce C.

18. A triangle has sides $a = 7$, $b = 9$, and an area of 18. Determine the measure of the included angle C.

19. Explain why the area formula $\frac{1}{2}ab\sin C$ is equivalent to $\frac{1}{2}$(base)(height) in a triangle. (You may produce a diagram on paper to support your explanation.)

20. A triangle has sides $a = (x + 2)$ and $b = (2x - 3)$ with an included angle of $30°$. Express the area of the triangle as a function of x.

Hard Questions

21. Prove that the formula Area $= \frac{1}{2}ab\sin C$ is equivalent to the traditional area formula Area $= \frac{1}{2}$(base)(height). Provide a step-by-step explanation.

22. A triangle has sides $a = 5$, $b = 7$, and an area of 8.75. Calculate the measure of the included angle C.

23. In a triangle, one side is given by $a = 3x$ and another by $b = (x + 4)$ with an included angle of $45°$. If the area is 15, solve for x.

24. Consider a triangle with sides $a = (2x + 1)$ and $b = (x + 3)$ and an included angle $60°$. If the area is 10, determine the value of x.

25. The area of a triangle is 20. Given that two sides are 8 and 10, find all possible measures for the included angle C (if any) that satisfy the area formula.

26. A triangle has sides $a = 10$ and $b = 11$, and its area is approximately 25. Calculate the included angle C to the nearest degree, and comment on its closeness to $30°$.

27. In a triangle, if $a = 7$, $b = 9$, and the area is 20, determine $\sin C$ and hence compute the measure of the included angle C.

28. A triangle has sides $a = (x + 4)$ and $b = (2x - 1)$ with an included angle $45°$. If the area is 30, determine the value of x.

29. The area of a triangle is given by $A = \frac{1}{2}(2x + 3)(x - 1)\sin C$. If $x = 3$ and the included angle $C = 30°$, compute the area of the triangle.

30. Using the area formula $\frac{1}{2}ab\sin C$, derive an expression for the altitude drawn to the side of length b in terms of a, b, and C. (Write your derivation in detail.)