ILLAWARRA TUTORING

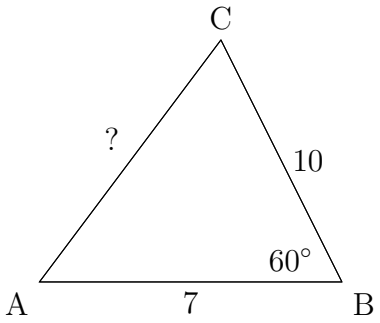
In this worksheet, you will master the techniques to calculate the area of a triangle using trigonometric methods. You will use the formula $A = \frac{1}{2}ab\sin C$ to solve a variety of problems, including numerical evaluations, algebraic expressions and word problems.

Easy Questions

1. Use the formula $A = \frac{1}{2}ab\sin C$ to find the area of a triangle with side lengths $a = 6$ and $b = 8$ when the included angle is $30°$. Explain your working.
2. A triangle has sides of lengths 10 and 15 with an included angle of $45°$. Calculate its area. Show all steps.
3. Determine the area of a triangle where $a = 5$, $b = 9$ and the included angle is $60°$. Write down your working clearly.
4. In a triangle, if $a = 5$, $b = 7$ and the area is 8.75, find the value of $\sin C$. Explain the steps used to obtain your answer.
5. Explain which angle is used in the formula $A = \frac{1}{2}ab\sin C$ and why it is important to consider the included angle when calculating the area of a triangle.

Intermediate Questions

6. The triangle ABC is shown in the diagram below.



   Calculate the area of $\triangle ABC$ given that the angle between the sides of lengths 7 and 10 is $60°$.

7. A triangular park has two sides measuring 12 m and 20 m with an included angle of $70°$. Find the area of the park. Show all working.

8. In a triangle the side $a = 8$ and the included angle is 45°. If the area of the triangle is 20, find the length of side b. Explain your process.

9. A triangle has sides $a = 9$ and $b = 11$ and an area of 42.75. Determine the measure of the included angle C. Include all steps.

10. A student calculated the area of a triangle with sides 6 and 8 and an included angle of 30° but obtained an area of 12. Identify and explain the error in the student's calculation.

11. State and explain the derivation of the formula $A = \frac{1}{2}ab\sin C$. Write your explanation clearly.

12. A triangle has two sides of lengths 5 and 8 with an area of 30. Determine $\sin C$ of the included angle and then calculate C in degrees. Show all working.

13. In a triangle, if $a = 3$, the included angle is 30° and the area is 15, find the length of side b. Explain your steps.

14. Calculate the area of a triangle with side lengths 7 and 9 and an included angle of 75°. (Use $\sin 75^\circ \approx 0.9659$). Detail your calculations.

15. Show that if you double the included angle in a triangle while keeping the adjacent sides constant, the area does not necessarily double. Use the area formula and explain your reasoning.

16. If $a = x + 2$ and $b = 2x$ with an included angle of 45°, write an expression for the area of the triangle in terms of x. Clearly show each step in your derivation.

17. A triangular ramp is constructed with sides measuring 4 m and 6 m, and the included angle is 80°. Calculate the area of the ramp. Provide a clear explanation.

18. A triangle has an area of $50\,\text{m}^2$ with one side length of 10 m and the other side b. If the included angle is 60°, determine the value of b. Explain your procedure.

19. Given a triangle with sides $a = 8$ and $b = 12$ and an included angle of 50°, compute its area to one decimal place using $\sin 50^\circ \approx 0.7660$. Show your working.

20. A triangle is drawn on grid paper with adjacent side lengths of 5 cm and 7 cm and an included angle of 65°. Calculate its area. Provide all calculations.

Hard Questions

21. Derive the formula $A = \frac{1}{2}ab\sin C$ for the area of a triangle using trigonometric principles. Provide a detailed step-by-step explanation.

22. A triangle has sides given by $a = 2x$ and $b = x + 3$ with an included angle of 30°. If the area of the triangle is 10, solve for x. Present your solution clearly.

23. In a triangle, let $a = 3k$ and $b = 4k$ with an included angle of 60°. Express the area in terms of k and then determine the value of k if the area is 12. Explain your working.

24. A triangle has two sides measuring 11 and 13. When the included angle is 90°, the area is maximised. If the actual area is 30% less than this maximum area, determine the measure of the actual included angle. Provide clear working.

25. Using the area formula $A = \frac{1}{2}ab\sin C$, prove that for a fixed product ab, the area is maximised when the included angle C is 90°. Write a clear and rigorous proof.

26. A triangle has sides $a = 5$ and $b = 12$ with an included angle of 40° (which is twice 20°). Calculate its area and then compute the area if the included angle were 90°. Compare the two results and explain your findings.

27. For a triangle with sides 7 and 10, determine the measure of the included angle C if the area is 30. Use the formula and justify each step in your solution.

28. Using pen and paper, construct a triangle with sides 6 and 8 and an included angle of 55°. Then, calculate the area of your constructed triangle. Write a brief description of your construction process and all calculations.

29. Given the area formula $A = \frac{1}{2}ab\sin C$, discuss how small changes in the included angle C affect the area when C is near 90°. Use basic calculus concepts (such as differentiation with respect to C) to support your discussion. Provide a clear explanation.

30. A triangular plot of land has sides of lengths 16 m and 22 m with an unknown included angle. If the area of the plot is $100\,\text{m}^2$, determine the value of $\sin C$ and then find the measure of C in degrees. Additionally, discuss any potential limitations or considerations when using the formula $A = \frac{1}{2}ab\sin C$ in practical situations. Provide full working and explanations.