ILLAWARRA TUTORING

In this worksheet you will learn to determine the area of a sector using your understanding of radians. Recall that the area of a sector of a circle with radius r and central angle θ (in radians) is given by

$$\text{Area} = \frac{1}{2}r^2\theta.$$

Easy Questions

1. Calculate the area of a sector with $r = 5$ and $\theta = \frac{\pi}{4}$.

$$\text{Area} = \frac{1}{2} \cdot 5^2 \cdot \frac{\pi}{4}.$$

2. Find the area of a sector if the radius is 10 and the central angle is $\frac{\pi}{3}$.

$$\text{Area} = \frac{1}{2} \cdot 10^2 \cdot \frac{\pi}{3}.$$

3. A sector of a circle has $r = 7$ and $\theta = \frac{\pi}{2}$. Calculate its area.

$$\text{Area} = \frac{1}{2} \cdot 7^2 \cdot \frac{\pi}{2}.$$

4. Convert an angle of $60°$ to radians, and then compute the area of a sector with $r = 4$ using the converted angle.

$$60° = \frac{\pi}{3}, \quad \text{Area} = \frac{1}{2} \cdot 4^2 \cdot \frac{\pi}{3}.$$

5. Determine the radius of a circle if a sector with $\theta = \frac{\pi}{6}$ has an area of 3π.

$$\frac{1}{2}r^2 \cdot \frac{\pi}{6} = 3\pi.$$

Intermediate Questions

6. A sector of a circle has an area of 2π and a central angle $\theta = \frac{\pi}{2}$. Find the radius.

$$\frac{1}{2}r^2 \cdot \frac{\pi}{2} = 2\pi.$$

7. Calculate the area of a sector with $r = 8$ and $\theta = \dfrac{2\pi}{3}$.

$$\text{Area} = \frac{1}{2} \cdot 8^2 \cdot \frac{2\pi}{3}.$$

8. A circle has a sector with $\theta = \dfrac{\pi}{5}$ and an area of 10. Find the radius.

$$\frac{1}{2}r^2 \cdot \frac{\pi}{5} = 10.$$

9. Write an expression for the area of a sector of a circle in terms of r and θ.

$$\text{Area} = \frac{1}{2}r^2\theta.$$

10. A sector has an area equal to one-fifth of the total area of its circle. If the central angle is θ, express θ in terms of π.

    Hint: Total area is πr^2, so set

$$\frac{1}{2}r^2\theta = \frac{1}{5}\pi r^2.$$

11. For a circle with $r = 12$, a sector has an area of 24π. Calculate the value of θ.

$$\frac{1}{2} \cdot 12^2 \cdot \theta = 24\pi.$$

12. The area of a sector with $r = 9$ is 20.25π. Find θ.

$$\frac{1}{2} \cdot 9^2 \cdot \theta = 20.25\pi.$$

13. If the central angle of a sector is doubled while the radius remains constant, by what factor does the area change? Explain your reasoning briefly.

    (Hint: The area is directly proportional to θ.)

14. An artist draws a sector (a "slice of pizza") with $r = 5$. If the area of the slice is 10, find the measure of the central angle in radians.

$$\frac{1}{2} \cdot 5^2 \cdot \theta = 10.$$

15. For a sector with $r = 3$, the central angle increases from $\dfrac{\pi}{6}$ to $\dfrac{\pi}{3}$. Calculate the increase in the area of the sector.

$$\Delta\text{Area} = \frac{1}{2} \cdot 3^2 \cdot \left(\frac{\pi}{3} - \frac{\pi}{6}\right).$$

16. Two sectors with the same area have radii 4 and 6, and central angles θ_1 and θ_2 respectively. Write an equation relating θ_1 and θ_2.

$$\frac{1}{2} \cdot 4^2 \cdot \theta_1 = \frac{1}{2} \cdot 6^2 \cdot \theta_2.$$

17. A student calculated the area of a sector with $r = 10$ and $\theta = \dfrac{\pi}{8}$ and obtained 10π. Determine if this answer is correct. If not, compute the correct area.

$$\text{Correct Area} = \frac{1}{2} \cdot 10^2 \cdot \frac{\pi}{8}.$$

18. Express the area of a sector in terms of the circle's circumference C rather than its radius.

(Hint: Use $r = \dfrac{C}{2\pi}$.)

19. Given a sector of area A with central angle θ in a circle of radius r, express r in terms of A and θ.

$$\frac{1}{2}r^2\theta = A.$$

20. A wheel has $r = 0.5$ and rotates through a central angle of $\dfrac{\pi}{3}$. Determine the area of the corresponding sector.

$$\text{Area} = \frac{1}{2} \cdot (0.5)^2 \cdot \frac{\pi}{3}.$$

Hard Questions

21. A circular track has $r = 100\,\text{m}$. A section corresponding to a central angle of $\dfrac{\pi}{10}$ is marked. Calculate the area of this sector.

$$\text{Area} = \frac{1}{2} \cdot 100^2 \cdot \frac{\pi}{10}.$$

22. In a circle with unknown radius, a sector with $\theta = 2$ has an area of 20. Write an equation and solve for r.

$$\frac{1}{2}r^2 \cdot 2 = 20.$$

23. A design requires a sector whose area is exactly 15% of the total circle. Express the central angle θ in radians in terms of π.

(Hint: Equate $\dfrac{1}{2}r^2\theta$ with $0.15\pi r^2$.)

24. A sector has an area of 12π, and its central angle is one quarter of the full circle. Find the radius of the circle.

$$\frac{1}{2}r^2 \cdot \left(\frac{2\pi}{4}\right) = 12\pi.$$

25. A mechanical component has a circular part of radius r. A sector with central angle θ is removed so that the remaining area is 80% of the original circle. Write an equation and solve for θ in terms of π.

$$\pi r^2 - \frac{1}{2}r^2\theta = 0.8\pi r^2.$$

26. A circular garden has a sector to be paved. If the paved area is $50\,\text{m}^2$ and $r = 5\,\text{m}$, determine the required central angle in radians.

$$\frac{1}{2} \cdot 5^2 \cdot \theta = 50.$$

27. A circular pizza of radius $12\,\text{cm}$ is cut into equal sectors. If one sector has an area of 24π, how many slices are there?

(Hint: Compare the total area $\pi \cdot 12^2$ with the sector area.)

28. A wheel is divided into two equal sectors per full rotation. If one sector has an area of $10\,\text{m}^2$, determine the radius of the wheel.

$$\frac{1}{2}r^2 \cdot \pi = 10.$$

29. The areas of two sectors with the same central angle are in the ratio $9 : 16$. What is the ratio of their radii?

(Hint: Recall that area is proportional to r^2.)

30. A circular disc has a sector with area 18, given by $\frac{1}{2}r^2\theta = 18$. If the radius is increased by 10% while keeping θ constant, by what percentage does the area of the sector increase? Show your calculation.