ILLAWARRA TUTORING

In this worksheet you will learn to determine the area of a sector using your understanding of radians. You will apply the formula

$$A = \frac{1}{2}r^2\theta$$

to solve a range of problems from easy to challenging.

Easy Questions

1. Calculate the area of a sector with radius 4 and central angle $\frac{\pi}{4}$ radians.
2. Find the area of a sector where the radius is 6 and the central angle is $\frac{\pi}{3}$ radians.
3. A sector has an area of 10 and a radius of 4. Determine the central angle in radians.
4. A circle has a radius of 5. Find the area of a sector with a central angle of $\frac{\pi}{2}$ radians.
5. Given that the area of a circle is 36π, compute the area of a sector with a central angle of 1 radian.

Intermediate Questions

6. Find the area of a sector with radius 10 and central angle 2 radians.
7. A sector has an area of 50 and a central angle of 1 radian. Determine the radius.
8. Find the central angle in radians of a sector with radius 3 and area 2.
9. A sector has an area of 15 and a central angle of $\frac{\pi}{5}$ radians. Calculate the radius.
10. A sector in a circle of radius 7 has an area of 12.25. Find the central angle in radians.
11. A sector of a circle has a radius of 9 and an area of 20.25. Determine the central angle in radians.
12. If the area of a sector is doubled while keeping the central angle constant, explain how the radius changes. Provide a mathematical justification.

13. If the central angle of a sector is doubled while keeping the radius constant, describe how the area of the sector changes. Explain your reasoning.

14. A circular clock has a radius of 15. If the hand sweeps an angle of $\frac{\pi}{6}$ radians, what is the area of the sector covered?

15. Find the area of a sector with radius 12 and a central angle of 0.25 radians.

16. The area of a sector is 3.5 square units and the central angle is 0.7 radians. Determine the radius.

17. A circular sector has a central angle of 1 radian. Express its area in terms of the radius r.

18. Compute the area of a sector of a circle with radius 8 and central angle 1.5 radians.

19. Determine the radius of a circle if a sector with a central angle of 2.5 radians has an area of 31.25.

Hard Questions

20. Prove that if two sectors of different circles have the same area and the same central angle, then the radii of the circles must be equal.

21. A circular sector has area A and central angle θ. Show that if the central angle increases by 10% while the radius remains constant, the area increases by 10%. Provide a justification for your answer.

22. A sector of a circle has an area of 18. If the central angle is increased by 25% and, as a consequence, the area increases to 22.5, determine the original radius given that the original central angle was 1 radian.

23. A circular sector with radius 13 has an area of 42.25. Find the central angle in radians and verify your answer.

24. Explain why the area of a sector is proportional to the square of its radius. Provide a real-world example where this quadratic relationship is significant.

25. A lollipop stick is cut into two pieces. One piece is used to form a complete circle and the other is used to form a circular sector with a fixed central angle of 1 radian. If the total stick length is fixed, discuss qualitatively how the area of the sector varies as more of the stick is allocated to it rather than to the circle.

26. Derive the formula
$$A = \frac{1}{2}r^2\theta$$
for the area of a sector by considering the proportion of the circle that the sector represents.

27. Given the formula for the area of a sector, provide an argument explaining why increasing the radius affects the area quadratically while increasing the central angle affects the area linearly.

28. Determine the central angle in radians for which the area of the sector is exactly one third of the area of the circle.

29. A circular sector has a perimeter of 28, which consists of its arc length and two radii. If the central angle is 1.5 radians, determine the area of the sector.