ILLAWARRA TUTORING

In this worksheet you will practise proving and using fundamental trigonometric identities to simplify and solve equations.

Easy Questions

1. Simplify the expression $\sin^2\theta + \cos^2\theta$.
2. Show that $\sin^2\theta = 1 - \cos^2\theta$.
3. Simplify the expression $\tan^2\theta + 1$ and state the resulting identity.
4. Simplify $1 - 2\sin^2\theta$ as far as possible.
5. Verify that $1 + \tan^2\theta = \sec^2\theta$.

Intermediate Questions

6. Prove that $\dfrac{1-\cos\theta}{\sin\theta} = \dfrac{\sin\theta}{1+\cos\theta}$.
7. Prove that $\sec\theta - \tan\theta = \dfrac{1-\cos\theta}{\sin\theta}$.
8. Prove that $(\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$.
9. Simplify the expression $(1+\sin\theta)^2 - \cos^2\theta$.
10. Prove that $\dfrac{2\tan\theta}{\sec\theta - 1} = \sec\theta + 1$.
11. Prove that $\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta$.
12. Verify that $\tan\theta + \cot\theta = \sec\theta\ \csc\theta$.
13. Show that $(1+\cos\theta)^2 - \sin^2\theta = 2\cos\theta(1+\cos\theta)$.
14. Prove that $1 + \cos\theta = 2\cos^2\left(\dfrac{\theta}{2}\right)$.
15. Prove that $\sin\theta = 2\sin\left(\dfrac{\theta}{2}\right)\cos\left(\dfrac{\theta}{2}\right)$.
16. Simplify $\dfrac{\sin^2\theta}{1+\cos\theta}$ as far as possible.

17. Prove that $\dfrac{1-\tan^2\theta}{1+\tan^2\theta} = \cos 2\theta$.

18. Prove that $\dfrac{2\tan\theta}{1-\tan^2\theta} = \tan 2\theta$.

19. Simplify $\dfrac{1-\cos\theta}{\sin\theta} + \dfrac{1+\cos\theta}{\sin\theta}$.

20. Prove that $\dfrac{\sin\theta}{1+\cos\theta} = \tan\left(\dfrac{\theta}{2}\right)$.

Hard Questions

21. Prove that $\cos^4\theta - \sin^4\theta = \cos 2\theta$.

22. Prove that $\dfrac{\sin^2\theta}{1-\cos\theta} + \dfrac{\cos^2\theta}{1+\cos\theta} = 2$.

23. Prove that
$$(1+\cos\theta)^3 - (1-\cos\theta)^3 = 2\cos\theta\left(3+\cos^2\theta\right).$$

24. Prove that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$.

25. Prove that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$.

26. Using a diagram if necessary, prove that
$$\sin\theta\,\sin\left(60^\circ-\theta\right)\,\sin\left(60^\circ+\theta\right) = \frac{1}{4}\sin 3\theta.$$

27. Prove that $(\tan\theta+\cot\theta)^2 = \sec^2\theta + \csc^2\theta$.

28. Prove that $4\cos^2\theta - 3 = \dfrac{\cos 3\theta}{\cos\theta}$ (for $\cos\theta \neq 0$).

29. Prove that $\cos\theta\,\csc\theta - \sin\theta\,\sec\theta = \cot\theta - \tan\theta$.

30. Prove that $\sin 2\theta\,\sin 3\theta + \cos 2\theta\,\cos 3\theta = \cos\theta$.