ILLAWARRA TUTORING

In this worksheet you will practise proving and using fundamental trigonometric identities to simplify expressions and solve equations. Work through the questions carefully and justify each step in your proofs.

Easy Questions

1. Simplify the expression $\sin^2\theta + \cos^2\theta$.
2. Prove that $1 + \tan^2\theta = \sec^2\theta$ for all values of θ for which both sides are defined.
3. Simplify the expression $\cos^2\theta - \sin^2\theta$ and state its equivalent double–angle form.
4. Write the trigonometric function $\tan\theta$ in terms of $\sin\theta$ and $\cos\theta$.
5. Rearrange the identity $\sec^2\theta - \tan^2\theta = 1$ to express $\tan^2\theta$ in terms of $\sec^2\theta$.

Intermediate Questions

6. Prove that $\sin(2\theta) = 2\sin\theta\cos\theta$ by using the definition of sine for the sum of two angles.
7. Prove that $\cos(2\theta) = 2\cos^2\theta - 1$ starting from the Pythagorean identity.
8. Prove that $\cos(2\theta) = 1 - 2\sin^2\theta$ using the identity $\sin^2\theta + \cos^2\theta = 1$.
9. Starting from $\sin^2\theta + \cos^2\theta = 1$, show that $1 - \cos^2\theta = \sin^2\theta$.
10. Express $\sin^2\theta$ in terms of $\cos(2\theta)$.
11. Express $\cos^2\theta$ in terms of $\cos(2\theta)$.
12. Prove that $\tan^2\theta + 1 = \sec^2\theta$ by dividing the Pythagorean identity by $\cos^2\theta$.
13. Prove that $1 + \cot^2\theta = \csc^2\theta$ by dividing the Pythagorean identity by $\sin^2\theta$.
14. Simplify the expression $\dfrac{1-\cos\theta}{\sin\theta}$ using an appropriate half-angle identity.
15. Simplify the expression $\dfrac{\sin^2\theta}{1+\cos\theta}$ to an equivalent expression involving $\sin\theta$.
16. Verify that $\dfrac{1-\sin\theta}{\cos\theta}$ is equivalent to $\dfrac{\cos\theta}{1+\sin\theta}$ by appropriate algebraic manipulation.

17. Simplify the expression $\dfrac{\sec\theta - \csc\theta}{\tan\theta - \cot\theta}$ by writing all functions in terms of $\sin\theta$ and $\cos\theta$.

18. Express $\dfrac{1+\cos\theta}{1-\cos\theta}$ in terms of $\tan\left(\dfrac{\theta}{2}\right)$.

19. Prove that $\sin\theta\cos\theta = \dfrac{1}{2}\sin(2\theta)$ by using a suitable trigonometric identity.

20. Simplify the expression $\dfrac{1+\tan^2\theta}{\sec^2\theta}$ to its simplest form.

Hard Questions

21. Prove that $\dfrac{1+\sin\theta}{1-\sin\theta} = \left(\dfrac{1+\tan\left(\frac{\theta}{2}\right)}{1-\tan\left(\frac{\theta}{2}\right)}\right)^2$ by expressing sine in terms of $\tan\left(\dfrac{\theta}{2}\right)$.

22. Prove that $\tan\theta + \cot\theta = 2\csc(2\theta)$ by writing both tangent and cotangent in terms of sine and cosine.

23. Simplify the expression $\dfrac{\sin\theta + \sin 3\theta - \sin 5\theta}{\cos\theta + \cos 3\theta - \cos 5\theta}$ to a single trigonometric function using sum-to-product formulas.

24. Prove that $\dfrac{1+\cos\theta - \sin\theta}{1+\cos\theta + \sin\theta} = \dfrac{1-\tan\left(\frac{\theta}{2}\right)}{1+\tan\left(\frac{\theta}{2}\right)}$ by converting the expressions using half-angle identities.

25. Prove the identity $\tan(3\theta) = \dfrac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$ starting from the angle addition formulas.

26. Prove that $\csc\theta - \cot\theta = \tan\left(\dfrac{\theta}{2}\right)$ by expressing the left-hand side in terms of half-angle functions.

27. Simplify the expression $\dfrac{1-\cos\theta - \sin\theta}{1-\cos\theta + \sin\theta}$ to an expression involving $\tan\left(\dfrac{\theta}{2}\right)$.

28. Prove that $(\sec\theta + \tan\theta)(\csc\theta + \cot\theta) = \sec\theta\csc\theta$ by writing all functions in terms of sine and cosine and simplifying.

29. Using half-angle identities, show that

$$\sin\theta = \frac{2\tan\left(\frac{\theta}{2}\right)}{1+\tan^2\left(\frac{\theta}{2}\right)} \quad \text{and} \quad \cos\theta = \frac{1-\tan^2\left(\frac{\theta}{2}\right)}{1+\tan^2\left(\frac{\theta}{2}\right)}.$$

30. Simplify the expression $\dfrac{1+\sin\theta + \cos\theta}{1-\sin\theta + \cos\theta}$ to an expression in terms of $\tan\left(\dfrac{\theta}{2}\right)$.