ILLAWARRA TUTORING

This worksheet will deepen your understanding of radian measure and its advantages over degrees in certain calculations. You will practise converting between degrees and radians, applying the arc length formula, and justifying why radian measure is the natural choice in many mathematical contexts.

Easy Questions

1. Convert 60° to radians.
2. Convert 90° to radians.
3. Convert $\frac{\pi}{4}$ radians to degrees.
4. Explain why using radians is advantageous when computing arc lengths, referring to the formula $s = r\theta$.
5. Convert 180° to radians.

Intermediate Questions

6. Convert 45° to radians.
7. Convert 135° to radians.
8. Convert 225° to radians.
9. Convert 315° to radians.
10. Convert $\frac{\pi}{6}$ radians to degrees.
11. Convert $\frac{2\pi}{3}$ radians to degrees.
12. A circle has radius 5. If an arc has length 4, find the angle in radians that subtends the arc.
13. For a circle of radius 3 with a central angle of $\frac{\pi}{2}$, find the length of the arc.
14. A circle of radius 7 has an arc of length 7. Determine the measure of the central angle in radians.
15. Demonstrate that an angle of 360° is equivalent to 2π radians.

16. Derive the arc length formula $s = r\theta$ and explain why the formula holds true specifically when θ is measured in radians.

17. A circle with radius 10 has an arc length of 5. Calculate the corresponding central angle in radians.

18. Explain why using degrees complicates arithmetic in arc length calculations compared to using radians.

19. If a sector in a circle of radius 4 has an area of 2, determine the central angle (in radians) that defines the sector.

20. Find the radian measure of an angle that forms a quarter of a circle.

Hard Questions

21. Prove that for any circle the radian measure of a central angle is given by the ratio $\theta = \frac{s}{r}$, where s is the arc length and r the radius.

22. For a circle of radius 8, show that if the arc length is also 8, then the central angle must be 1 radian.

23. A wheel with radius 0.5 produces an arc length of 3.14. Calculate the angle swept (in radians) and comment on the result.

24. For a circle of radius 5 and a central angle of 3 radians, determine the chord length. Use the formula $c = 2r\sin\left(\frac{\theta}{2}\right)$, and express your answer in terms of sine.

25. An arc of a circle has a length equal to one-third of the circle's circumference. Find the measure of the corresponding central angle in radians, and explain your approach.

26. Derive the formula for the area of a sector, $A = \frac{1}{2}r^2\theta$, clearly explaining each step.

27. For a circle with radius r, if the original central angle θ (in radians) is increased by 10% resulting in an arc length increase of 5, express the new central angle in terms of r and θ. (Explain your reasoning.)

28. A pendulum of fixed length swings through an angle of θ radians. If the angle is doubled, discuss how the arc length traced by the pendulum changes. Provide a quantitative explanation.

29. Prove that if $s = r\theta$, then $\frac{ds}{d\theta} = r$. Explain each step of your proof.

30. Discuss why radian measure is considered the natural parameter for curves in calculus. Provide examples to support your explanation.