ILLAWARRA TUTORING

In this worksheet you will deepen your understanding of radian measure and its advantages over degrees in certain calculations. You will practise converting between degrees and radians, applying the arc length and sector area formulas, and exploring the natural relationship between a circle's geometry and radian measure.

## Easy Questions

1. Convert $45°$ to radians. Answer by writing your answer in terms of $\pi$.
2. Convert $\frac{\pi}{2}$ radians to degrees.
3. Write the radian measure of a full circle and a half circle.
4. A circle has a radius $r = 3$. Calculate the length of an arc subtended by a central angle of $\frac{\pi}{3}$ radians using the formula $s = r\theta$.
5. In a few sentences, explain one advantage of using radian measure rather than degrees in calculations involving circles.

## Intermediate Questions

6. Convert $120°$ to radians. Express your answer in terms of $\pi$.
7. Convert $270°$ to radians.
8. Convert $\frac{\pi}{3}$ radians to degrees.
9. A circle has a radius $r = 5$. Find the length of the arc corresponding to the central angle $\frac{\pi}{4}$ radians.
10. A circle with radius $r = 7$ has an arc of length 7. Find the central angle in radians.
11. Calculate the area of a sector in a circle with radius 4 and central angle $\frac{\pi}{3}$ radians using the formula $A = \frac{1}{2}r^2\theta$.
12. Simplify the expression $\frac{5\pi}{6} + \frac{\pi}{3}$.
13. Determine the radian measure of a quarter of a circle.
14. Find a coterminal angle between 0 and $2\pi$ for an angle of $\frac{5\pi}{4}$ radians.

15. Calculate the difference between $\frac{3\pi}{2}$ and $\frac{\pi}{6}$ radians.

16. An arc of a circle has length 10 and is subtended by a central angle of $\frac{\pi}{2}$ radians. Find the radius of the circle.

17. Evaluate $\frac{\pi}{2} - \frac{\pi}{6}$.

18. Given that a semicircle measures $\pi$ radians, calculate the radian measure of three-quarters of a circle.

19. A circle has a circumference of 20. First, find its radius, and then determine the length of an arc corresponding to a central angle of 2 radians.

20. For a circle with radius $r = 3$ and a central angle of 4 radians, verify that the arc length is given by $s = r\theta$, and compute its value.

# Hard Questions

21. Prove that for a unit circle, the length of an arc is equal to the measure of its central angle in radians.

22. Derive the formula $A = \frac{1}{2}r^2\theta$ for the area of a sector given that $s = r\theta$. Explain each step in your derivation.

23. A circular track has a radius of 50. A runner completes an arc of length 75. First, determine the central angle in radians, and then convert this angle to degrees.

24. Prove that the radian measure of a full circle is equal to $2\pi$ by using the relationship between the circumference and the radius.

25. In a short paragraph, describe an advantage of radian measure when deriving formulas for arc length and sector area, compared to using degree measure.

26. A circle is divided into $n$ equal sectors. Express the measure of each sector's central angle in radians and show that the sum of these angles is $2\pi$.

27. A sector of a circle has an area of 10 and a central angle of 2 radians. Determine the radius of the circle and then compute the length of the arc for this sector.

28. Two circles have different radii. If both have a central angle of 1 radian, compare the ratio of the arc length to the circumference for each circle. Explain your reasoning.

29. For a fixed central angle measured in radians, discuss qualitatively how the arc length and the area of the corresponding sector vary as the radius increases.

30. A wheel with radius 0.3 rotates through an angle of 3 radians. Calculate the distance travelled along the circumference of the wheel, and explain the significance of using radian measure in this calculation.