ILLAWARRA TUTORING

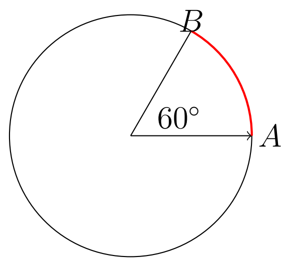
This worksheet is designed to deepen your understanding of radian measure and its advantages over degrees in certain calculations. You will practise converting between degrees and radians, applying radian measure to compute arc lengths and sector areas, and exploring some theoretical advantages of using radian measure.

Easy Questions

1. Convert 60° to radians.
2. Convert 180° to radians.
3. Express 270° in radians.
4. Convert $\frac{\pi}{6}$ to degrees.
5. What is the radian measure of a full rotation?

Intermediate Questions

6. Convert 45° to radians.
7. An angle measures $\frac{2\pi}{3}$ radians. Convert it to degrees.
8. A circle has a radius of 5. Calculate the length of the arc subtended by a central angle of 60°. First convert the angle to radians.



9. For a circle with a radius of 10, compute the area of a sector with a central angle of 90°. Be sure to convert the angle to radians before applying the formula.
10. Express 385° in radians in its simplest form.
11. Determine the radian measure corresponding to 0° and explain briefly why it is unique.

12. Convert $-30°$ to radians.

13. A circular track has a circumference of 400 metres. Using the radian measure concept, find the central angle subtended by an arc of length 50 metres.

14. If an angle measures $\frac{7\pi}{8}$ radians, what is its approximate value in degrees? (Give your answer to one decimal place.)

15. Write $\frac{11\pi}{12}$ in a mixed form if possible, expressing your answer in terms of π.

16. If an arc on a circle has a length of 3 and the radius is 2, calculate the measure of the central angle in radians.

17. Given $\theta = \frac{\pi}{3}$, express this angle in degrees.

18. For the angle $\frac{5\pi}{2}$ radians, determine a coterminal angle between 0 and 2π.

19. Express $\frac{-3\pi}{4}$ radians as a positive coterminal angle between 0 and 2π.

20. If an arc length of a circle is $\sqrt{2}$ and the radius is 1, determine the corresponding central angle in degrees.

Hard Questions

21. Explain why the radian measure of an angle is independent of the circle's radius. Include a brief proof based on the definition of radian measure.

22. Derive the result that a full rotation in a circle measures 2π radians by using the formula for the circumference of the circle.

23. Derive the formula for the area of a sector with central angle θ radians in a circle of radius r. Explain how using radian measure simplifies the derivation.

24. Two circles have radii r_1 and r_2, and arcs of lengths s_1 and s_2 respectively, subtend the same central angle θ. Prove that $\frac{s_1}{r_1} = \frac{s_2}{r_2}$.

25. A sector of a circle has an area of 5 square units when the central angle is $\frac{\pi}{3}$ radians. Determine the radius of the circle.

26. A circle has a circumference of 12π. If an arc length equals one third of the circumference, calculate its central angle in both radians and degrees.

27. An angle of $59°$ is approximated by $\frac{\pi}{3}$ radians. Calculate the error of this approximation and express your answer in degrees.

28. An ant walks along an arc of a circle with radius 3 metres, subtending an angle of $\frac{2\pi}{5}$ radians. Compute the distance walked and discuss why radian measure is a natural choice in such problems.

29. A sector of a circle has an area of 8 square units and a central angle of θ radians. Derive an expression for the radius in terms of θ, and then compute the radius when $\theta = \dfrac{4\pi}{5}$.

30. Consider two circles with radii r_1 and r_2. A fixed angle θ (in radians) subtends arcs on both circles. Derive the relationship between the areas of the corresponding sectors in the two circles and explain the benefit of using radian measure in making such comparisons.