In this worksheet you will learn to sketch and interpret the graphs of $\sin x$, $\cos x$ and $\tan x$ functions. You will be guided through plotting key points, recognising periodic behaviour and understanding the unique characteristics of each function.

Easy Questions

1. Sketch the graph of $y = \sin x$ over the interval $[0, 2\pi]$. Identify the key points at $x = 0,\ \pi/2,\ \pi,\ 3\pi/2$, and 2π.
2. Sketch the graph of $y = \cos x$ over the interval $[0, 2\pi]$ and mark its maximum and minimum points.
3. State the period of the function $y = \sin x$ and briefly explain your answer.
4. Sketch the graph of $y = \tan x$ over one period, indicating at least one complete cycle.
5. For $y = \tan x$, determine the locations of the vertical asymptotes in the interval $(-\pi/2, \pi/2)$.

Intermediate Questions

6. List the coordinates of the key points of $y = \sin x$ on the interval $[0, 2\pi]$.
7. Sketch the graph of $y = \cos x$ over $[0, 2\pi]$ and label its intercepts with the x-axis.
8. Determine the x-intercepts of $y = \cos x$ within one period $[0, 2\pi]$.
9. Explain in your own words why the functions $y = \sin x$ and $y = \cos x$ are regarded as periodic.
10. Identify the type of symmetry exhibited by the graph of $y = \cos x$ and explain briefly how you recognised it.
11. Sketch $y = \tan x$ for one period and clearly label the vertical asymptotes.
12. List the x-values within $(-\pi/2, \pi/2)$ where $y = \tan x$ is undefined.
13. Compare the graphs of $y = \sin x$ and $y = \cos x$ by describing the horizontal shift that relates them.
14. Using the graph of $y = \sin x$, indicate on the interval $[0, 2\pi]$ where the function is increasing and where it is decreasing.

15. Explain how the periodic nature of $y = \tan x$ is reflected in its graph.

16. For $y = \cos x$ on $[0, 2\pi]$, determine its maximum and minimum values and the corresponding x-values.

17. Identify the domain of $y = \tan x$ over one period, stating the values that must be excluded.

18. Explain, with reference to its vertical asymptotes, how the end behaviour of $y = \tan x$ is demonstrated.

19. Sketch the graphs of $y = \sin x$ and $y = \cos x$ on the same set of axes over $[0, 2\pi]$. Identify and label the points where they intersect.

20. Describe how the graph of $y = \sin x$ demonstrates periodic behaviour, and state how this pattern repeats.

Hard Questions

21. Explain in detail the horizontal shift that relates $y = \sin x$ and $y = \cos x$, and discuss how this is evident from their graphs.

22. Determine the exact x-values in the interval $[0, 2\pi]$ where $y = \tan x$ changes sign, and explain your reasoning.

23. Sketch the graph of $y = \sin x$ over $[0, 2\pi]$ on your paper. Emphasise the zero crossings, maximum and minimum points, and note the intervals of increase and decrease.

24. Based on the graph of $y = \cos x$, explain how its even symmetry can be used to predict its behaviour in intervals outside $[0, 2\pi]$.

25. Describe the repeating pattern observed in the graph of $y = \tan x$ over two consecutive periods.

26. Sketch a graph of $y = \tan x$, and label the key features including one period, the zero, and the vertical asymptotes.

27. Using your sketch of $y = \sin x$, identify the intervals in $[0, 2\pi]$ where the function is monotonically increasing and where it is monotonically decreasing. Justify your answer.

28. From the graph of $y = \cos x$, discuss the significance of its maximum value in relation to the overall shape of the curve. How does this affect the graph's appearance?

29. Compare the intervals during which $y = \sin x$ is increasing or decreasing with those of $y = \cos x$. Explain how and why these intervals differ.

30. Synthesize your understanding by describing how the periodic characteristics of $y = \sin x$, $y = \cos x$ and $y = \tan x$ can be identified from their graphs. Discuss how these features are useful in real world applications that involve periodic phenomena.