ILLAWARRA TUTORING

This worksheet will help you learn to sketch and interpret the graphs of $\sin(x)$, $\cos(x)$ and $\tan(x)$, enabling you to visualise periodic behaviour. Work through each question carefully and produce clear sketches where required.

Easy Questions

1. Sketch a simple graph of $\sin(x)$ for $x \in [0, 2\pi]$. Label the points where the function is zero, reaches its maximum, and minimum.
2. Sketch a simple graph of $\cos(x)$ for $x \in [0, 2\pi]$. Clearly label the maximum, minimum and the x-intercepts.
3. List the key coordinates (with exact values) for one period of $\sin(x)$ over $[0, 2\pi]$. Provide the coordinates of the intercepts and extrema.
4. Sketch the graph of $\tan(x)$ for $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Indicate the vertical asymptotes and the x-intercept.
5. Write down the x-values of the vertical asymptotes for $\tan(x)$ within one period.

Intermediate Questions

6. Sketch the graph of $\sin(x)$ over $x \in [-2\pi, 2\pi]$. Mark clearly the x-intercepts.
7. Sketch the graph of $\cos(x)$ over $x \in [-2\pi, 2\pi]$. Label at least two maximum points and two minimum points.
8. Using your knowledge of the $\sin(x)$ graph, write down all the x-coordinates in $[0, 2\pi]$ where the function is zero, and where it attains its maximum and minimum.
9. Sketch $\tan(x)$ for $x \in [-\pi, \pi]$. Label the vertical asymptotes and at least one point on each branch.
10. On the graph of $\cos(x)$ for $x \in [0, 2\pi]$, indicate all points where $\cos(x) = \frac{1}{2}$. State the exact x-values.
11. Draw a sketch of both $\sin(x)$ and $\cos(x)$ on the same set of axes for $x \in [0, 2\pi]$. Label the points where the two graphs intersect.
12. On a sketch of $\sin(x)$ over one period, label and write down its amplitude and period.

13. Sketch a close-up of the $\tan(x)$ graph near one of its vertical asymptotes (either $\frac{\pi}{2}$ or $-\frac{\pi}{2}$). Explain briefly the behaviour of the function as it approaches the asymptote.

14. On the same set of axes for $x \in [0, 2\pi]$, sketch the graphs of $\sin(x)$, $\cos(x)$, and (where defined) $\tan(x)$. Briefly describe one key difference in the behaviour of $\tan(x)$ compared to the other two functions.

15. For the graph of $\sin(x)$ on $[0, 2\pi]$, identify the interval(s) where the function is increasing and the interval(s) where it is decreasing. Explain your reasoning briefly.

16. On the graph of $\cos(x)$ over $[0, 2\pi]$, mark the intervals where $\cos(x)$ is increasing and where it is decreasing.

17. Plot the graphs of $\sin(x)$ and $\tan(x)$ on the same axes for $x \in \left(0, \frac{\pi}{2}\right)$. Describe in a sentence how the growth of $\tan(x)$ compares with that of $\sin(x)$ as x increases.

18. Using the graph of $\cos(x)$ on $[0, 2\pi]$, determine the exact x-values for which $\cos(x) = \frac{1}{2}$. Explain how these are identified on the graph.

19. Sketch the graph of $\sin(x)$ over one period clearly showing the progression from the start of the cycle to the end. Label the start, maximum, midpoint, minimum and the end of the cycle.

20. Examine the graph of $\cos(x)$ over $[0, 2\pi]$. Write a short explanation of how you can identify the maximum, minimum, and x-intercepts from the graph.

Hard Questions

21. Sketch the graph of $\sin(x)$ over two complete periods (for example, $x \in [0, 4\pi]$). Label all intercepts and indicate the maximum and minimum points.

22. Sketch the graph of $\cos(x)$ over $x \in [-3\pi, 3\pi]$. Indicate the symmetry of the graph and explain briefly what this symmetry is.

23. Sketch a detailed graph of $\tan(x)$ over $x \in (-\pi, \pi)$. Make sure to include the vertical asymptotes and at least two key points on each branch.

24. Given a sketch of $\sin(x)$, explain in a short paragraph how you would determine its period and frequency. Include the definition of period in your explanation.

25. Sketch both $\sin(x)$ and $\cos(x)$ on the same set of axes for $x \in [0, 2\pi]$. Compare the two graphs by describing one similarity and one difference between them.

26. Analyse the relationship between $\sin(x)$ and $\tan(x)$ by sketching both functions on a single set of axes over an interval where they are both defined (for instance, $x \in [0, \frac{\pi}{4}]$). Explain how their behaviours differ as x increases.

27. For the graph of $\cos(x)$ over one period, determine and label the maximum point, minimum point, and the x-intercepts. Write a brief explanation of how you found these points.

28. Discuss the symmetry properties of $\sin(x)$, $\cos(x)$ and $\tan(x)$. In your answer, mention which functions are even or odd and explain what that implies graphically. Include sketches to support your explanation.

29. Sketch the graph of $\tan(x)$ with an emphasis on the behaviour near the vertical asymptotes. Explain in a short paragraph why these asymptotes occur.

30. Using the basic graphs of $\sin(x)$, $\cos(x)$ and $\tan(x)$ as references, write a short explanation on how you would predict the overall behaviour (periodicity, intercepts, extrema) of a periodic function that does not involve any transformations. Support your explanation with simple sketches.