This worksheet is designed to help you learn to sketch and interpret the graphs of $\sin x$, $\cos x$, and $\tan x$. You will visualise periodic behaviour by plotting key points, identifying asymptotes and recognising symmetry in these functions.

Easy Questions

1. Sketch the graph of $y = \sin x$ for one complete cycle. Use a horizontal axis marked in multiples of π and indicate the intercepts, maximum and minimum points.

2. Sketch the graph of $y = \cos x$ for one complete cycle. Clearly label the intercepts, maximum, and minimum points.

3. Sketch the graph of $y = \tan x$ between its two nearest vertical asymptotes. Mark the points where the function crosses the horizontal axis.

4. Identify the key features (intercepts, maximum and minimum points) on the graph of $y = \sin x$ drawn over one period.

5. Label the intercepts and turning points on the graph of $y = \cos x$, indicating where the function reaches its maximum and minimum.

Intermediate Questions

6. Plot the graph of $y = \sin x$ on a set of axes for $0 \leq x \leq 2\pi$. Label all intercepts, the maximum at $x = \pi/2$, and the minimum at $x = 3\pi/2$.

7. Plot the graph of $y = \cos x$ on a grid for $0 \leq x \leq 2\pi$. Mark the intercepts, maximum, and minimum points.

8. Plot the graph of $y = \tan x$ in the interval $-\frac{\pi}{2} < x < \frac{\pi}{2}$. Clearly indicate the vertical asymptotes and the point where the function passes through the origin.

9. Explain why the graph of $y = \tan x$ has vertical asymptotes by discussing the behaviour of $\sin x$ and $\cos x$ as x approaches $\frac{\pi}{2}$ and $-\frac{\pi}{2}$.

10. Given the standard graph of $y = \sin x$, determine its period and explain your reasoning.

11. For the graph of $y = \cos x$, identify the point in the interval $[0, 2\pi]$ where the function reaches its maximum value and justify your answer.

12. On a coordinate system, draw a rough sketch of $y = \tan x$, highlighting its periodic nature and marking at least one complete cycle between asymptotes.

13. Explain the periodic behaviour observed in the graph of $y = \sin x$. In your explanation, include the concept of period and mention the value of one complete cycle.

14. Explain why $\sin x$ and $\cos x$ are considered periodic functions. Provide definitions and refer to their repeating patterns.

15. Compare the periodic behaviour of $y = \sin x$ and $y = \cos x$. Discuss their similarities and any differences in the positioning of their key points.

16. Determine the period of $y = \tan x$ by analysing its graph. Describe how the spacing between vertical asymptotes relates to the period.

17. Sketch the graphs of $y = \sin x$ and $y = \cos x$ on the same set of axes for $0 \leq x \leq 2\pi$. Identify and label the points where the two graphs intersect.

18. Describe the behaviour of $y = \tan x$ as it approaches its vertical asymptotes. What does the graph indicate about the function's value near these asymptotes?

19. Explain how the graph of $y = \cos x$ can be used to infer its increasing and decreasing intervals. Support your explanation with reference to the curve's turning points.

20. Using a sketch of $y = \sin x$, show that the function repeats its pattern every 2π units. Include labelled key points in your sketch.

Hard Questions

21. Analyse the graph of $y = \sin x$ and provide a mathematical justification for why its period is 2π. Include a discussion on the repeating nature of its key features.

22. Using the graph of $y = \cos x$, demonstrate how the function is symmetric about the y-axis. Provide a clear explanation of this even symmetry.

23. Provide a detailed explanation of the behaviour of $y = \tan x$ near its vertical asymptotes. In your answer, illustrate your explanation with a hand-drawn graph and reference the limits as x approaches the asymptote.

24. Explain how the graph of $y = \sin x$ can be used to solve the equation $\sin x = 0$ over a given interval. Describe the steps involved in interpreting the graph to find the solutions.

25. Analyse the relationship between $y = \sin x$ and $y = -\sin x$ by sketching their graphs on the same axes. Explain the transformation that relates the two graphs.

26. Outline the process of sketching the graph of $y = \cos x$. In your answer, describe how you determine the key points and the overall shape of the graph.

27. Using the graph of $y = \tan x$, describe how you would determine its period based on the spacing between its vertical asymptotes. Provide a detailed explanation.

28. Discuss how the graphical representation of $y = \sin x$ can aid in real-world applications. Describe a scenario involving periodic phenomena and explain how the graph is used to model this behaviour.

29. Interpret the graph of $y = \cos x$ by identifying its intervals of increase and decrease. Provide a justification for your observations based on the curvature and turning points.

30. Critically analyse the graph of $y = \tan x$, discussing the implications of its asymptotic behaviour on the concept of limits. Explain how the graph illustrates the idea that the function does not have finite limits at the asymptotes.