ILLAWARRA TUTORING

In this worksheet you will learn how to apply trigonometric functions to model periodic phenomena you encounter in real life.

Easy Questions

1. The function $f(t) = 5\cos(t)$ models a simple periodic phenomenon. Identify the amplitude and period of the function.
2. The function $f(t) = \sin(2t)$ represents a periodic process. Determine its period.
3. The function $f(t) = 3\sin\left(t - \frac{\pi}{4}\right)$ provides a model for a periodic event. Calculate $f(0)$.
4. A student claims that the function $y = \sin(t)$ can be used to model the temperature variation during the day. Explain one advantage and one limitation of using a trigonometric model in this context.
5. Using pen and paper, draw the graph of $y = \cos(t)$ for one complete period. Clearly label the key points.

Intermediate Questions

11. Consider the tide height given by $f(t) = 2\cos\left(\frac{\pi}{6}t - \frac{\pi}{3}\right) + 4$. Determine the period and the vertical shift of this function.
12. A model for daylight hours is given by $f(d) = 3\sin\left(\frac{\pi}{182.5}d\right) + 12$, where d is the day number. Determine the period and amplitude of the function and explain what these parameters represent in the context of daylight variation.
13. A Ferris wheel ride is modelled by $f(t) = 5\sin\left(\frac{\pi}{15}t\right) + 6$, where t is time in seconds. Calculate the time taken for one complete revolution.
14. A sound wave is represented by $f(t) = 0.01\sin(2\pi\, 1000t)$. Determine the frequency of this sound in Hertz.
15. A pendulum's displacement is approximated by $f(t) = 0.2\cos\left(\frac{\pi}{2}t\right)$. Explain the significance of the period and amplitude in the context of the pendulum's motion.

16. A seasonal temperature is modelled by $T(m) = 10\sin\left(\frac{2\pi}{12}m - \frac{\pi}{2}\right) + 20$, where m is the month (with $m = 1$ representing January). Identify the amplitude and phase shift and explain what each represents.

17. A tidal model is given by $f(t) = 3\cos\left(\frac{\pi}{7}t\right)$, where t is in hours. Find the period of the tide cycle.

18. A school uses a rotating class schedule that repeats every 8 hours. Write a general sine function of the form $f(t) = A\sin(wt + \phi) + B$ that has maximum magnitude 1 and no vertical shift. Clearly state your function.

19. Given $f(t) = 4\sin\left(\frac{\pi}{4}t\right) + 3$ which models a biological rhythm with an expected period of 8, verify that the period is indeed 8 and discuss the effect of the vertical shift.

20. A satellite's orbit is modelled by $f(t) = 500\cos\left(\frac{2\pi}{90}t\right) + 700$, where t is measured in minutes. Determine the period of the orbit and explain what the vertical shift represents.

21. Given a model of the form $f(t) = A\cos(wt + \phi) + B$ for a periodic phenomenon, observations show that the maximum value is 10 and the minimum value is 2. Determine the values of A and B.

22. If the period of $f(t) = A\sin(wt + \phi) + B$ is observed to be 24, determine the corresponding value of w in its simplest form.

23. A river's water level oscillates as per $f(t) = 3\sin\left(\frac{\pi}{12}t - \frac{\pi}{4}\right) + 5$. Find the first positive time t at which the water level reaches its maximum, given that $t = 0$ is the start of the observation.

24. For the function $f(t) = 4\cos\left(\frac{\pi}{8}(t - 2)\right) + 6$, explain how you would determine the time at which the function reaches its minimum value.

25. A model for daylight hours is given by $f(t) = 2\sin\left(\frac{2\pi}{365}(t - 81)\right) + 12$. Estimate the number of daylight hours on day $t = 172$.

Hard Questions

21. A buoy in the ocean oscillates vertically such that it reaches a maximum height of 7 m above mean sea level and a minimum height of 1 m, completing one full cycle in 12 s. Derive a sinusoidal function of the form $f(t) = A\cos(wt + \phi) + B$ that models the buoy's motion, given that at $t = 0$ the buoy is at its mean position and rising.

22. A town experiences a periodic temperature variation described by $T(t) = A\sin(wt + \phi) + B$, where the highest temperature of 35°C occurs on day 200 and the lowest of 5°C on day 20. Assuming a 365-day cycle, determine the values of A, B, w, and ϕ. Provide detailed reasoning.

23. The sound from a tuning fork is modelled by $f(t) = A\sin(wt + \phi)$ with a frequency of 512 Hz and an amplitude of 0.005 m. Write an explicit expression for $f(t)$ given that at $t = 0$ the wave is at equilibrium and increasing.

24. Consider the coastal water temperature modelled by $f(t) = 4\cos\left(\frac{\pi}{10}(t-3)\right) + 8$, where t is in hours. Determine the time at which the temperature is decreasing most rapidly.

25. A pendulum's angular displacement is given by $f(t) = A\cos(wt + \phi)$. It is observed that the pendulum reaches its maximum displacement of 0.1 radians at $t = 2$ s and its minimum displacement of -0.1 radians at $t = 4$ s. Determine an explicit function $f(t)$ that models the pendulum's motion.

26. A satellite's elliptical orbit causes its distance from Earth to vary sinusoidally. If the satellite is at 950 km (perigee) at $t = 0$ and at 1050 km (apogee) after 45 minutes, write a function of the form $f(t) = A\cos(wt + \phi) + B$ that models its distance from Earth as a function of time t in minutes.

27. A model for the variation in daylight hours is given by $f(d) = A\sin\left(\frac{2\pi}{365}(d - \phi)\right) + B$, where d is the day number. If the longest day has 15 h of daylight (on day 172) and the shortest has 9 h (on day 355), determine the values of A, B, and ϕ.

28. A coastal city's tide level is represented by $f(t) = A\cos(wt + \phi) + B$. If the tide rises from 2 m to 8 m in 6 h and then falls back to 2 m in the next 6 h, and the highest tide occurs at $t = 3$ h, derive the function $f(t)$.

29. A variable star's brightness is modelled by $f(t) = A\sin(wt + \phi) + B$. Observations show that its brightness varies between 12 and 18 magnitudes over a period of 5 days, with the brightness increasing at $t = 1$ day. Find a possible function for $f(t)$.

30. A sound engineer designs a system where the amplitude of a sinusoidal signal controls a motor's speed. The signal is described by $f(t) = A\cos(wt + \phi) + B$, with a period of 0.5 s. If the maximum value of the signal is 15 and the minimum is 5, and the signal is increasing at $t = 0$, determine an explicit expression for $f(t)$.