ILLAWARRA TUTORING

In this worksheet you will learn how to apply trigonometric functions to model periodic phenomena encountered in real life. Work through each question carefully and show all your working.

Easy Questions

1. State the amplitude and period of the function $y = \sin x$.
2. For the function $y = 2\sin x$, state its amplitude and period.
3. A tide height is modelled by $y = 5\sin\left(\frac{\pi}{12}t\right)$. Determine the period of this function.
4. A Ferris wheel's height is given by $h(t) = 2\sin\left(\frac{\pi}{8}t\right) + 5$. Find the maximum and minimum heights reached by a rider.
5. If the tide height is modelled by $y = 3\cos\left(\frac{\pi}{6}t\right) + 10$, what is the time interval for one complete cycle?

Intermediate Questions

6. A Ferris wheel is modelled by $h(t) = 4\sin\left(\frac{\pi}{15}t - \frac{\pi}{2}\right) + 6$, where t is in seconds. Determine the amplitude, period, phase shift, and midline of the function.
7. A lighthouse beam's intensity is given by $I(t) = 10 + 2\cos\left(\frac{\pi}{6}t\right)$ with t in minutes. Determine the period of this function and the range of possible intensities.
8. The seasonal temperature variation is modelled by $T(d) = 20 + 5\sin\left(\frac{2\pi}{365}(d - 80)\right)$ where d is the day number. Find the period and determine on which day the temperature is at the average value.
9. Given the pendulum displacement $d(t) = 7\sin\left(\frac{\pi}{3}t\right)$, find the first time $t > 0$ when the displacement reaches its maximum.
10. Determine the midline and amplitude of the function $y = -3\sin\left(\frac{\pi}{4}t\right) + 8$.
11. Daylight hours in a year are modelled by $H(m) = 12 + 1.5\sin\left(\frac{\pi}{182.5}(m - 100)\right)$ where m is the day number. Determine the period of the function and the maximum number of daylight hours.

12. A sound wave is represented by $S(t) = 0.8\cos(100\pi t)$. Compute the period of this wave.

13. The water level in a harbour varies according to $L(t) = 6\sin\left(\frac{\pi}{4}t + \frac{\pi}{6}\right) + 12$. Find the phase shift of this function.

14. A vibrating string's displacement is given by $s(t) = 0.02\sin(100\pi t)$. Identify the frequency and period of the vibration.

15. A daily biorhythm is modelled by $R(t) = 3\cos\left(\frac{\pi}{12}t - \frac{\pi}{3}\right) + 7$. Determine the vertical shift and the time shift (phase shift) of the cycle.

16. Consider the vibration $v(t) = A\sin\left(\frac{\pi}{5}t + \frac{\pi}{2}\right)$ where the displacement at $t = 0$ is 5. Find the value of A.

17. The tide height is given by $h(t) = 2\sin\left(\frac{\pi}{12}t\right) + 8$. Compute the height at $t = 6$ hours.

18. A pendulum's displacement is modelled by $f(t) = 4\sin\left(\frac{\pi}{2}t\right) + 4$. Determine the period and the time interval between successive maximum displacements.

19. Meteorologists model atmospheric pressure by $P(t) = 1013 + 5\cos\left(\frac{\pi}{4}t - \frac{\pi}{8}\right)$. Compute the phase shift and the maximum pressure.

20. Given the function $F(t) = 10\sin\left(\frac{\pi}{6}t + \frac{\pi}{3}\right) + 20$ that models population oscillations, determine its amplitude and period.

Hard Questions

21. A Ferris wheel rotates once every 40 seconds and a rider's height is modelled by $H(t) = 2.5\cos\left(\frac{\pi}{20}t\right) + 10$. Prove that the period of this function is 40 seconds and determine the height of the rider after 10 seconds.

22. A tide cycle is described by $T(t) = 4\sin\left(\frac{\pi}{12}t - \frac{\pi}{4}\right) + 12$ where t is in hours. Show that the tide reaches its first peak when $t = 9$ hours and determine the corresponding maximum height.

23. A sound wave is given by $S(t) = 0.5\cos\left(100\pi t - \frac{\pi}{4}\right)$. Compute its angular frequency, frequency, and period.

24. The daylight hours in a town vary periodically with a period of 365 days. Construct a sinusoidal model for the daylight hours if the maximum is 15 hours and the minimum is 9 hours, and the maximum occurs on day 172. State the amplitude, midline, period, and phase shift in your model.

25. A city's weekly traffic is modelled by $V(t) = A\sin\left(\frac{2\pi}{168}t + \phi\right) + D$ where t is in hours. Suppose the maximum traffic volume of 1200 vehicles per hour occurs at

$t = 8$ and the minimum of 800 vehicles per hour occurs at $t = 20$ on the same day. Determine the values of A, D, and ϕ in your model.

26. An oscillating bridge deck has its displacement described by $D(t) = 0.3\sin\left(\frac{\pi}{2}t - \frac{\pi}{6}\right)$ (in metres), where t is in seconds. Determine the time interval between successive zero displacements and explain your reasoning.

27. A buoy's vertical displacement is modelled by $B(t) = 1.2\cos\left(\frac{\pi}{3}t + \frac{\pi}{3}\right)$ (in metres). Find the time when the buoy first reaches its lowest point after $t = 0$ and determine that lowest displacement.

28. The seasonal river flow is given by $R(t) = 50\sin\left(\frac{2\pi}{12}(t-2)\right) + 300$ (in cubic metres per second), where t is in months. Determine the amplitude and period of the flow. Then, explain how the model's behaviour would change if the function were shifted horizontally by 3 months.

29. A pendulum clock is designed so that its displacement is given by $x(t) = 0.5\sin\left(\frac{\pi}{1.5}t + \frac{\pi}{4}\right)$. Verify that the period of the pendulum is 3 seconds and find the displacement at $t = 1.5$ seconds.

30. A scientist models the concentration of a chemical in a reactor with the function $C(t) = 0.8\sin\left(\frac{\pi}{4}t + \frac{\pi}{8}\right) + 2$, where t is in minutes. Determine the time when the concentration first reaches its maximum value and compute that maximum concentration.