ILLAWARRA TUTORING

This worksheet focuses on applying trigonometric functions to model periodic phenomena encountered in everyday life. You will work on real-world modelling problems using sine and cosine functions.

Easy Questions

1. Name a real-life phenomenon that exhibits periodic behaviour and can be modelled using a trigonometric function.
2. Write an example of a sine function that can be used to model the variation in daily temperature. For example, use a formula of the form $f(t) = A + B\sin\left(\omega t + \phi\right)$.
3. Consider the tide model given by $f(t) = 3 + 2\sin\left(\frac{\pi}{6}t\right)$, where t is the time in hours. Write down the amplitude, period and midline of this function.
4. A phenomenon has a period of 24 hours. Write the expression for the angular frequency ω in terms of the period.
5. If a periodic event repeats every 10 seconds, express the corresponding angular measure in radians for one complete cycle.

Intermediate Questions

6. A coastal tide reaches a high of 6 m and a low of 2 m. High tide occurs at 15:00. Assuming the tide follows a sine model, write an equation $f(t)$ (with t in hours after midnight) that models this tide.
7. During the year the number of daylight hours varies between 9 hours in winter and 15 hours in summer, with maximum daylight on day 172. Construct a cosine function $f(d)$ (with d being the day number) that models the daylight hours.
8. A Ferris wheel with a radius of 5 m rotates fully every 40 seconds. A passenger starts at the lowest point. Write a sine function $f(t)$ (with t in seconds) that models the passenger's height above the ground assuming the centre is at 5 m above the ground.
9. A sound wave has a frequency of 440 Hz. Write an expression for its displacement using a sine function in the form $f(t) = \sin\left(2\pi \cdot 440\,t\right)$, where t is in seconds.

10. The temperature during a day is modelled by $f(t) = 20 + 5\cos\left(\frac{\pi}{12}(t-6)\right)$, where t is the time in hours. Calculate the temperature at 18:00.

11. For the tide model in question 6, determine the amplitude and the midline of the function.

12. Using the Ferris wheel model from question 8, state the period of the function.

13. For the sound wave in question 9, calculate the period and express your answer in milliseconds.

14. Consider the function $f(t) = 10 + 4\sin\left(\frac{\pi}{9}t\right)$. Determine the maximum and minimum values of $f(t)$.

15. For the temperature model in question 10, compute the temperature at midnight (i.e. at $t = 0$).

16. A function of the form $f(t) = a + b\sin\left(\omega(t-c)\right)$ has a maximum of 30 at $t = 14$ and a minimum of 10 at $t = 2$. Determine the values of a and b.

17. For the daylight hours model in question 7, determine the amplitude of the variations.

18. For the tide model $f(t) = 4 + 3\sin\left(\frac{\pi}{6}(t-2)\right)$, solve for t when $f(t) = 7$. Show your working.

19. The function $f(t) = 5 + 3\cos\left(\frac{\pi}{8}t\right)$ has its maximum at $t = 0$. Modify the function by introducing a phase shift so that its maximum occurs at $t = 4$. Write the new function.

20. Using the Ferris wheel model from question 8, determine the passenger's height at $t = 10$ seconds.

Hard Questions

21. A coastal tide reaches a high of 8 m and a low of 2 m. High tide occurs at 5:00 and the period is 12 hours. Derive a sine function $f(t)$ with t in hours after midnight that models this tide. Clearly show how you determine the amplitude, midline, angular frequency and phase shift.

22. The average annual temperature is 15°C. The maximum temperature of 25°C occurs on day 200 and the minimum of 5°C on day 20. Determine a cosine function $f(d) = 15 + A\cos\left(\omega(d-c)\right)$ modelling the temperature, where d is the day of the year. Explain your reasoning.

23. A Ferris wheel has its centre 10 m above the ground and a radius of 8 m. It completes one revolution every 4 minutes. A rider starts at the lowest point. Derive a sine function $f(t)$ (with t in minutes) for the height of the rider above ground, detailing your calculations.

24. A sound wave is given by $f(t) = A\sin\Big(360t + \phi\Big)$, where t is in seconds. If the frequency is 360 Hz and the wave passes through zero in the upward direction at $t = 0$, and its maximum displacement is 0.01 m, determine the amplitude A and the phase shift ϕ.

25. For the daylight model $f(d) = 12 + 3\cos\left(\frac{2\pi}{365}(d - 173)\right)$, solve for the day d when the daylight lasts 15 hours. Provide full working.

26. A temperature model is given by $f(t) = 18 + 6\sin\left(\frac{\pi}{12}(t - c)\right)$. If the maximum temperature occurs at $t = 16$, determine the phase shift c.

27. A pendulum's angular displacement is modelled by $\theta(t) = 20\sin\left(\frac{\pi}{3}t + \phi\right)$ (in degrees). If the maximum displacement occurs at $t = 0.5$ seconds, find the value of ϕ.

28. Sketch a diagram of a Ferris wheel showing the centre, the radius, and the positions corresponding to maximum and minimum heights. Label the key points clearly.

29. An economic index is modelled by $f(t) = 1000 + 50\sin\left(\frac{2\pi}{5}(t - 1)\right)$, where t is in years. Explain what the amplitude, phase shift and vertical shift represent in this context. Also, determine the length of one complete economic cycle.

30. A pendulum's angular displacement is measured to be 15° at $t = 1$ s, -15° at $t = 3$ s and 0° at $t = 2$ s. Derive a sine function of the form $f(t) = A\sin\Big(\omega(t - c)\Big)$ to model the pendulum's motion. Clearly explain your process.