ILLAWARRA TUTORING

In this worksheet, you will analyse how changes in amplitude, period, and phase shift affect the graphs of trigonometric functions. You will answer a series of questions ranging from basic identification to more complex analysis and real-world applications. Remember to show all your working where necessary.

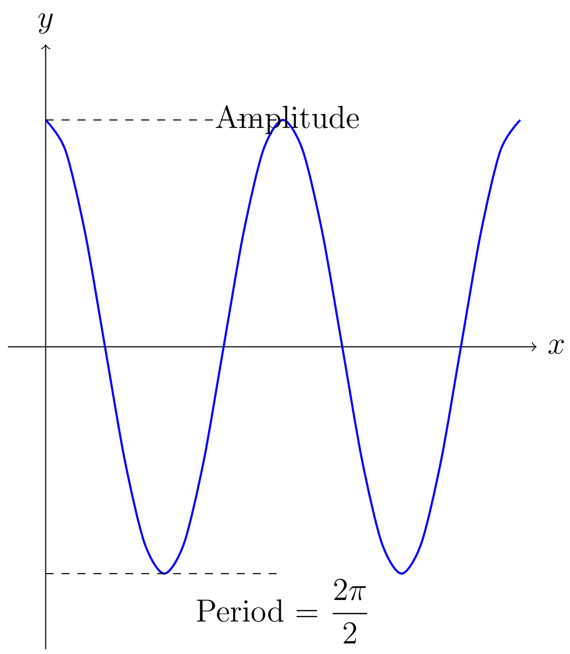
Easy Questions

1. State the definition of amplitude and determine the amplitude of $y = 2\sin x$.
2. Define the period of a trigonometric function and state the period of $y = \sin x$.
3. Explain what the phase shift represents and find the phase shift of $y = \sin\left(x - \frac{\pi}{4}\right)$.
4. Determine the amplitude, period, and phase shift of $y = -3\cos x$.
5. Sketch the graph of $y = 2\sin x$ on your graph paper. Ensure you label the key points such as the maximum, minimum, and intercepts.

Intermediate Questions

6. Describe how the graph of $y = 4\sin x$ differs from that of $y = \sin x$ in terms of amplitude.
7. Calculate the period of $y = \sin\left(2x\right)$ and explain your reasoning.
8. Determine the phase shift of $y = \cos\left(x - \frac{\pi}{2}\right)$.
9. Rewrite $y = -3\sin\left(3x+\pi\right)$ in the form $y = A\sin\left(Bx-C\right)$ and state its amplitude, period, and phase shift.
10. Determine the amplitude, period, and phase shift of $y = 2\cos\left(0.5x - \frac{\pi}{4}\right)$.
11. Compare the graphs of $y = \sin x$ and $y = 2\sin x$. Describe how the change in amplitude affects the graph.
12. Compare the graphs of $y = \sin x$ and $y = \sin(2x)$. Describe the effect on the period.
13. Compare the graphs of $y = \sin x$ and $y = \sin\left(x - \frac{\pi}{2}\right)$ and explain how the phase shift changes the graph.

14. Study the diagram below and label the amplitude and period of the function $y = 3\cos(2x)$.



15. Find the amplitude, period, and phase shift of $y = -2\cos\Big(4x - \pi\Big)$.

16. Explain how a negative amplitude affects the graph of a sine function.

17. Describe what happens to the graph of $y = \cos x$ when the period is halved.

18. Sketch the graph of $y = 5\sin\left(3x + \frac{\pi}{3}\right)$ on your graph paper, clearly marking the amplitude, period, and phase shift.

19. Determine the first positive value of x within one period for which $y = \sin\left(2x - \frac{\pi}{3}\right)$ reaches its maximum.

20. Describe in detail how the graph of $y = \sin x$ transforms into the graph of $y = 3\sin\Big(4x - \pi\Big)$ by discussing the changes in amplitude, period, and phase shift.

Hard Questions

21. Derive the formula for the period of a function in the form $y = \sin(Bx)$ and explain each step of your derivation.

22. Find the value of B for $y = \sin(Bx)$ if the function has a period of $\frac{\pi}{2}$, and explain how this value affects the graph.

23. Show that $y = -\cos\left(2x + \frac{\pi}{3}\right)$ can be expressed in the form $y = \cos\Big(2x + \theta\Big)$ for an appropriate phase shift θ. Determine the value of θ.

24. Provide a detailed analysis of the function $y = 7\sin\left(3x - \frac{\pi}{2}\right)$ by stating its amplitude, period, and phase shift. Then, sketch a labelled graph of the function on graph paper.

25. Determine the function that results when the graph of $y = \sin x$ is shifted to the right by $\frac{\pi}{6}$ and vertically stretched by a factor of 4.

26. Consider the function $y = 2\sin\left(5x + \frac{\pi}{4}\right)$ and determine the x-coordinate of the first positive zero after the phase shift has been applied.

27. Consider a sine function in the form $y = A\sin(Bx - C)$ that attains its maximum at $x = \frac{\pi}{4}$ and its minimum at $x = \frac{3\pi}{4}$. Assuming $A > 0$ and $B > 0$, deduce possible values for A, B, and C and justify your answers.

28. Show that the function $y = 4\cos(6x)$ has a period of $\frac{\pi}{3}$ by using the property $\text{Period} = \frac{2\pi}{B}$.

29. Determine the amplitude, period, and phase shift of $y = -2\sin\left(8x + \frac{\pi}{2}\right)$. In your response, explain the effect of the negative coefficient on the graph.

30. Design a real-world scenario where the concepts of amplitude, period, and phase shift of a trigonometric function are applied. Explain your reasoning and describe how changes in these parameters affect the scenario.