In this worksheet you will analyse how changes in amplitude, period, and phase shift affect the graphs of trigonometric functions. You will be asked to find these characteristics from function expressions and to explain or sketch the resulting transformations.

Easy Questions

1. Write in plain text the definition of amplitude for a sine or cosine function. Then state the amplitude of $f(x) = 3\sin(x)$.
2. Write in plain text the definition of period in the context of a trigonometric function. Then state the period of $f(x) = \sin(x)$.
3. Write in plain text the definition of phase shift for a trigonometric function.
4. Identify the amplitude and period of $f(x) = 2\cos(x)$.
5. Determine the period of $f(x) = \cos(2x)$.

Intermediate Questions

6. Determine the amplitude, period, and phase shift of $f(x) = 2\sin\left(x - \frac{\pi}{6}\right)$.
7. Sketch the graph of $y = -\sin(x)$ on your graph paper. In plain text, explain how the negative sign affects the graph compared to $y = \sin(x)$.
8. For the function $f(x) = 3\cos(2x)$, determine the amplitude, period, and phase shift. Then, sketch its graph on your graph paper.
9. Compute the phase shift of $f(x) = \sin\left(2\left(x - \frac{\pi}{4}\right)\right)$ and describe in plain text what it means for the graph.
10. Determine the amplitude and period of $f(x) = \cos(4x)$.
11. For $f(x) = -4\sin\left(x - \frac{\pi}{3}\right)$, find the amplitude, period, and phase shift.
12. The standard sine function $y = \sin(x)$ is transformed by a vertical stretch and a horizontal shift. Write the function that has been stretched vertically by a factor of 2 and shifted to the right by $\frac{\pi}{2}$. Then sketch the graph on your graph paper.

13. In plain text, explain how altering the amplitude of a trigonometric function affects its maximum and minimum values.

14. Write a short explanation on the effect of halving the period of $y = \sin(x)$ on its graph.

15. Write the function that represents a phase shift of $\frac{\pi}{3}$ to the left applied to $y = \sin(x)$.

16. Determine the amplitude, period, and phase shift of $f(x) = -2\cos(3x + \pi)$. (Hint: Factor to express in the form $A\cos\Big(3\Big(x - D\Big)\Big)$.)

17. Draw a diagram that illustrates a cosine function with amplitude 3, period 2π, and a phase shift of $\frac{\pi}{4}$ to the right. Label the maximum and minimum points as well as one complete cycle.

18. A table shows selected values of $y = \sin(x)$ and a transformed function $y = k\sin\Big(b\Big(x - d\Big)\Big)$. In plain text, describe how you can determine the amplitude and phase shift from comparing the two sets of values.

19. Consider the function $f(x) = \sin\left(x + \frac{\pi}{2}\right)$. In plain text, explain whether this represents a phase shift of the basic sine function and justify your answer.

20. Write a brief explanation discussing how amplitude and phase shift independently affect the graph of a trigonometric function.

Hard Questions

21. Consider a function of the form $f(x) = a\sin(bx - c)$ that has a maximum value of 4 and a minimum value of -4, and completes one full cycle in π. Assuming a, b, and c are positive, determine the values of a, b, and c. (Hint: The amplitude is 4, and use $\frac{2\pi}{b} = \pi$. Then use the phase shift condition corresponding to a typical sine maximum.)

22. Prove that the functions $f(x) = \sin(x)$ and $g(x) = \sin\Big(x - \pi\Big)$ are related by a phase shift. Determine the magnitude and direction of the phase shift and explain its effect on the graph.

23. Write $f(x) = -3\sin\left(2x - \frac{\pi}{2}\right)$ in the form $A\sin\Big(B\Big(x - D\Big)\Big)$ where $A > 0$. Determine the values of A, B, and D, and explain each step in your transformation.

24. A function of the form $g(x) = k\sin(bx - c)$ has its maximum at $x = \frac{\pi}{3}$ and has a period of $\frac{2\pi}{b}$. Assuming $k > 0$, $b > 0$, and $c > 0$, determine one set of possible values for k, b, and c. (Hint: For a standard sine function, the maximum occurs when the argument is $\frac{\pi}{2}$.)

25. Derive the formula for the period of $f(x) = \sin(bx)$, where $b > 0$, and then compute the period when $b = 5$.

26. For the function $f(x) = 4\cos\left(x + \frac{\pi}{4}\right)$, determine its amplitude and phase shift. Write your answer in plain text.

27. The function $g(x) = -5\sin\left(x - \frac{\pi}{8}\right)$ represents a vertically reflected sine wave that is shifted. Determine the amplitude, period, and phase shift of $g(x)$ and explain how each value affects the graph.

28. In plain text, provide a detailed explanation on how to determine the phase shift from the standard form $f(x) = \sin(Bx - C)$. Include any necessary algebraic steps.

29. Two graphs are given: $f(x) = \sin(x)$ and $g(x) = \sin\left(2\left(x - \frac{\pi}{6}\right)\right)$. Compare their periods and phase shifts, explaining the differences in their transformations.

30. A Ferris wheel's height above ground level is modelled by the function $h(t) = A\sin\left(Bt - C\right)$, where t is measured in seconds. If the Ferris wheel takes 60 seconds to complete one rotation and the maximum height reached is 30 metres (assume no vertical shift), determine appropriate values for A, B, and one possible value for C. In plain text, describe what each parameter represents in the context of the Ferris wheel.