ILLAWARRA TUTORING

In this worksheet you will analyse how changes in amplitude, period, and phase shift affect the graphs of trigonometric functions. For each question, determine the transformation parameters and explain how they affect the graph.

Easy Questions

1. For $y = \sin x$, find the amplitude, period, and phase shift.
2. For $y = 3\sin x$, determine the amplitude, period, and phase shift.
3. For $y = \sin 2x$, find the amplitude, period, and phase shift.
4. For $y = \sin\left(x - \frac{\pi}{3}\right)$, state the amplitude, period, and phase shift.
5. Sketch the graph of $y = 2\sin x$ for one period. Label the amplitude and indicate the period on your graph.

Intermediate Questions

6. For $y = -4\sin 2x$, determine the amplitude, period, and explain the effect of the negative sign.
7. For $y = 3\cos\left(x + \frac{\pi}{6}\right)$, find the amplitude, period, and phase shift.
8. Compare the graphs of $y = 2\sin x$ and $y = 2\sin\left(x - \frac{\pi}{4}\right)$. Describe the horizontal translation between them.
9. Explain how the graph of $y = \sin x$ is transformed to produce $y = 2\sin\left(x + \frac{\pi}{2}\right)$.
10. Rewrite $y = \sin\left(2x - \pi\right)$ in the form $y = \sin\left(2\left(x - \frac{\pi}{2}\right)\right)$ and state the amplitude, period, and phase shift.
11. Create a table of key points for the function $y = 2\cos\left(3x + \frac{\pi}{2}\right)$ over one period. List at least 5 points.
12. For $y = -\sin\left(x - \frac{\pi}{2}\right)$, determine the amplitude, period, and phase shift, and explain the effect of the negative sign on the graph.
13. List, in order, the three transformations applied to $y = 5\cos\left(2x - \frac{\pi}{3}\right)$.

14. Express $y = 3\sin\left(\frac{1}{2}x - \frac{\pi}{4}\right)$ in the form $y = 3\sin\left(\frac{1}{2}\left(x - \frac{\pi}{2}\right)\right)$ and state the amplitude, period, and phase shift.
15. Write a cosine function with amplitude 2, period 8π, and phase shift π to the right.
16. Decide if the following statement is true or false and explain your answer: Changing the amplitude of a sine function does not affect its period.
17. Construct a sine function with amplitude 1, period π, and a left phase shift of $\frac{\pi}{4}$.
18. Explain how the graph of $y = 2\cos x$ is altered when it is modified to $y = 2\cos\left(x - \frac{\pi}{2}\right)$.
19. Compare the graphs of $y = \sin x$ and $y = \sin\left(x + \pi\right)$. What phase shift occurs?
20. Describe the order in which transformations are applied in the function $y = a\sin\left(bx - c\right)$ and explain how each parameter (a, b, and c) affects the graph.

Hard Questions

21. Rewrite $y = -3\sin\left(4x + \pi\right)$ in the form $y = -3\sin\left(4\left(x + \frac{\pi}{4}\right)\right)$ and determine its amplitude, period, and phase shift.
22. Transform $y = 2\cos\left(3x - \frac{\pi}{2}\right)$ into the form $y = 2\cos\left(3\left(x - \frac{\pi}{6}\right)\right)$ and state the amplitude, period, and phase shift.
23. Rewrite $y = -2\sin\left(2\left(x + \frac{\pi}{6}\right)\right)$ in the equivalent form $y = -2\sin\left(2x + \frac{\pi}{3}\right)$ and identify the amplitude, period, and phase shift.
24. Construct a sine function with amplitude 4, period 3π, and a phase shift of $-\frac{\pi}{3}$ (shifted to the left).
25. Analyse how the graph of $y = 2\sin x$ is altered when changed to $y = 2\sin\left(2x\right)$. Identify the new period.
26. Prove that adding a constant inside the argument of a sine function (i.e. replacing x with $x - c$) results in a phase shift without changing the amplitude or period.
27. Compare the graphs of $y = \sin x$ and $y = -\sin x$. Explain the effect of the negative sign on the amplitude and the graph's orientation.
28. For $y = 3\sin\left(2x - \frac{\pi}{4}\right)$, determine the coordinates of the first three key points of one period after applying the phase shift. Sketch these points on your own graph (using pen and paper).
29. Discuss how the period and phase shift parameters in $y = a\sin\left(bx - c\right)$ can be adjusted independently without affecting the amplitude. Provide examples in your explanation.