In this worksheet you will learn to represent events using set notation and Venn diagrams, making complex relationships clearer.

Easy Questions

1. Write the set notation (using roster form) for the set of natural numbers less than 5.
2. Given $A = [1, 2, 3]$ and $B = [3, 4, 5]$, write the union $A \cup B$ in set notation.
3. Draw a Venn diagram representing two sets A and B, indicating the overlapping region.
4. Express in set notation the set of even numbers between 1 and 10 (inclusive) using roster form.
5. Given $A = [2, 4, 6, 8]$ and $B = [4, 8, 12, 16]$, write the intersection $A \cap B$ in set notation.

Intermediate Questions

6. Represent in set notation (using roster form) the set of odd numbers between 10 and 20.
7. Let $A = [2, 4, 6, 8, 10, 12, 14, 16, 18, 20]$ (multiples of 2 between 1 and 20) and $B = [3, 6, 9, 12, 15, 18]$ (multiples of 3 between 1 and 20). Write $A \cap B$ in set notation.
8. For $A = [1, 2, 3, 4, 5]$ and $B = [4, 5, 6, 7, 8]$, write the union $A \cup B$ in set notation.
9. With the same sets as in Q8, express the set difference $A - B$ in set notation.
10. Draw a Venn diagram showing two overlapping sets A and B. Label the regions corresponding to A, B, $A \cap B$, and $A \cup B$.
11. Given the universal set $U = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]$ and $A = [2, 4, 6, 8, 10]$, write the complement A^c in set notation.
12. For $A = [1, 2, 3, 4]$ and $B = [3, 4, 5, 6]$, express the symmetric difference (elements in either set but not in both) in set notation.
13. Write the set builder notation for the set A which consists of numbers greater than 0 and less than 5.

14. Let
$$A = [x : x \text{ is a vowel in the English alphabet}]$$
and
$$B = [x : x \text{ is a letter in the word (example)}].$$
Write $A \cap B$ in set notation.

15. Let $A = [x : x$ is a multiple of 4 and $1 \leq x \leq 20]$. List the elements of A in roster form.

16. Translate the statement "The set A consists of all elements that are not in B" into set notation using complement.

17. Given $A = [1, 2, 3, 4, 5]$ and $B = [3, 4, 5, 6, 7]$, express the set of elements in A but not in B in set notation.

18. If U is the set of all letters in the English alphabet and A is the set of vowels, write the complement A^c in set notation.

19. Using set builder notation, define the set of prime numbers between 1 and 20.

20. For arbitrary subsets A and B of a universal set U, express $A^c \cup B^c$ in set notation and state in words what this set represents.

Hard Questions

21. Let A, B, and C be sets. Write, in set notation, the set of elements that belong to exactly two of these sets.

22. Using pen and paper, draw a Venn diagram for three sets and clearly indicate the regions corresponding to elements that are in exactly two of the sets. Label all regions appropriately.

23. Prove using set theory laws that the complement of the union of two sets is equal to the intersection of their complements; that is, show that $(A \cup B)^c = A^c \cap B^c$. Provide a written proof.

24. Let
$$A = [x \in \mathbb{N} : x \text{ is a multiple of } 2, 1 \leq x \leq 20]$$
and
$$B = [x \in \mathbb{N} : x \text{ is a multiple of } 3, 1 \leq x \leq 20].$$
List the elements of $A \cap B$ and $A \cup B$ in roster form.

25. Suppose $U = [1, 2, 3, \ldots, 30]$, let A be the set of integers divisible by 4, and B be the set of integers divisible by 5. Using set notation, find $A \cap B$ and explain your reasoning.

26. Let $A = [x : x$ is an even number$]$ and $B = [x : x$ is a perfect square$]$. Express in set notation the set of numbers that are even perfect squares between 1 and 50.

27. Define $A = [x \in \mathbb{N} : x \text{ is prime}]$ and $B = [x \in \mathbb{N} : x > 2]$. Write the set difference $B - A$ in set builder notation.

28. Let U be the universal set of students in a class. If $A = [\text{students who play soccer}]$ and $B = [\text{students who play basketball}]$, express in set notation the set of students who play neither sport.

29. If A, B, and C are subsets of U, express in set notation the set of elements that belong to exactly one of the three sets.

30. State and illustrate (with a pen and paper diagram) De Morgan's laws for three sets. Then provide, in set notation, the expressions corresponding to these laws.