ILLAWARRA TUTORING

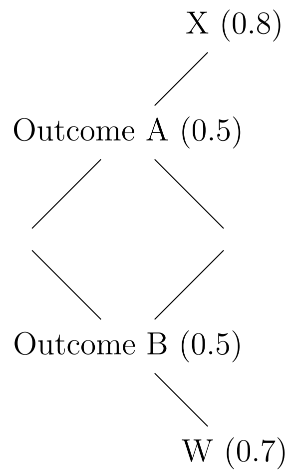
In this worksheet you will learn to use tree diagrams as a visual tool to represent sequences of events and calculate their probabilities. You will work through questions of increasing complexity.

Easy Questions

1. Draw a tree diagram representing two successive independent events. Let the first event have outcomes with probabilities 0.6 and 0.4. For the second event, each outcome has probabilities 0.7 and 0.3. (Use pen and paper to draw the diagram.)
2. Using the tree diagram from question 1, calculate the probability of the outcome that follows the 0.6 branch in the first event and the 0.3 branch in the second event.
3. A fair coin is tossed twice. Draw a tree diagram with outcomes heads and tails (each with probability 0.5) for each toss. Then calculate the probability of obtaining heads on both tosses.
4. A tree diagram shows the first event with two outcomes: one branch is labelled 0.8; the other branch is not labelled. Determine the missing probability and explain your reasoning.
5. Construct a tree diagram for two successive events where the first event has outcomes with probabilities 0.3 and 0.7, and the second event (following each outcome) has probabilities 0.4 and 0.6. Then, calculate the probability of following the branch with 0.7 then 0.4.

Intermediate Questions

6. A bag contains red and blue marbles. The probability to draw a red marble is 0.3 and a blue marble is 0.7. The marble is replaced before a second draw. Construct a tree diagram and compute the probability of drawing a red marble then a blue marble.
7. Consider the following tree diagram:



Calculate the probability of the branch that follows Outcome B and then outcome W.

8. Construct a tree diagram for two successive events. Let the first event have outcomes A and B with probabilities 0.5 each. If event A occurs, the second event has outcomes X (0.3) and Y (0.7); if event B occurs, the second event has outcomes X (0.6) and Y (0.4). Compute the probability of the outcome A then Y.

9. Draw a tree diagram for three successive independent events where each event has two outcomes with probabilities 0.5 each. Calculate the probability of the outcome sequence (first branch, then second branch, then first branch).

10. In a certain town, the probability that it is sunny on a given day is 0.7 and rainy is 0.3. If it is sunny, the probability of having a picnic is 0.6 and not having a picnic is 0.4. If it is rainy, the probability of having a picnic is 0.1 and not having a picnic is 0.9. Draw a tree diagram and find the probability that on a randomly chosen day a picnic is held.

11. A tree diagram shows that for the first event outcome C the branch probabilities for the second event are 0.35 and an unknown value. Determine the missing probability if the probabilities must sum to 1 and explain your reasoning.

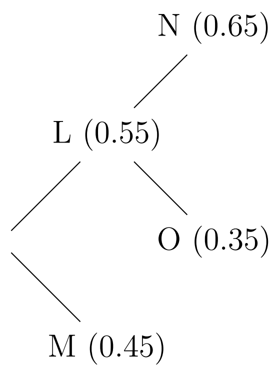
12. A process is represented by a tree diagram with two stages. The first stage has outcomes D (0.4) and E (0.6). For outcome D, the second stage outcomes are F (0.5) and G (0.5). Calculate the overall probability of following the branch D then F.

13. A student attempts three true-false questions. The probability of answering each question correctly is 0.6 independently. Construct a tree diagram and determine the probability that the student answers all three questions correctly.

14. In a two-stage process, the first event has outcomes H (0.25) and I (0.75). If H occurs, the second event has outcomes J (0.8) and K (0.2). If I occurs, the second event has outcomes J (0.4) and K (0.6). Compute the probability that outcome J occurs.

15. A person decides whether to take an umbrella based on the weather forecast. If the forecast predicts rain (with probability 0.4) then the person takes an umbrella with probability 0.9; if the forecast predicts no rain (with probability 0.6), the person takes an umbrella with probability 0.2). Construct a tree diagram and find the overall probability that the person takes an umbrella.

16. Consider the following tree diagram:



Calculate the probability of following the branch L then O.

17. In a two-stage process, the first event has outcomes P (0.4) and Q (0.6). Both outcomes lead to a common final event R in the second stage with probabilities 0.3 (after P) and 0.5 (after Q). Draw a tree diagram and calculate the overall probability of event R occurring.

18. A box contains two types of batteries. The probability of choosing a rechargeable battery is 0.2 and non-rechargeable is 0.8. If a rechargeable battery is chosen, the probability it lasts more than 5 hours is 0.9; if a non-rechargeable battery is chosen, the probability it lasts more than 5 hours is 0.4. Draw a tree diagram and find the probability that a randomly chosen battery lasts more than 5 hours.

19. Consider a process where an event occurs in three stages. The first stage has outcomes R (0.7) and S (0.3). If R occurs, the second stage has outcomes T (0.6) and U (0.4); if S occurs, the second stage has outcomes T (0.2) and U (0.8). For both T and U, assume the third stage has outcomes V (0.5) and W (0.5). Draw the tree diagram and determine the probability of the branch R then U then V.

20. In a two-stage process, the first event has outcomes X (0.65) and its complement. For outcome X, the second event outcomes are Y (0.4) and its complement. Draw a tree diagram. First, determine the missing probabilities. Then, compute the probability that the final outcome is the complement of Y after outcome X.

Hard Questions

21. Consider a three-stage process where all events are independent. The probabilities for the outcomes at each stage are as follows: Stage 1: A (0.5) and B (0.5).
Stage 2 if A: C (0.6) and D (0.4); if B: C (0.3) and D (0.7).
Stage 3 for all previous outcomes: E (0.8) and F (0.2).

Draw a complete tree diagram and calculate the probability of the outcome sequence B then D then E.

22. A process is described by a tree diagram where the first event has outcomes G (0.55) and its complement. For outcome G, the second event outcomes are H (0.65) and its complement. Determine the missing probabilities and compute the overall probability of obtaining G then H.

23. An incomplete tree diagram is given for a two-stage process. The first branch has probability I = 0.45. For this branch, one of the two outcomes in the second stage is given as 0.7. (a) Determine the missing probability on the second branch. (b) Then, compute the overall probability for the branch I then the missing outcome. (Show your steps.)

24. In a three-stage process, each stage has two outcomes. The probability of a desirable outcome in any stage is 0.3 and the undesirable is 0.7. (Assume independence.) Use a tree diagram to calculate the probability that the desirable outcome occurs at least once.

25. A factory has a machine that produces parts. The probability that a part is defective is 0.1. If a part is defective, it is reworked and has a 0.5 chance to become non-defective. If the part is non-defective, it is not reworked. Construct a tree diagram for a part going through production and potential rework. Compute the overall probability that a part leaving the process is non-defective.

26. A machine undergoes a three-stage quality check. In the first check, the probability to pass is 0.85. If it passes, the second check has a pass probability of 0.9. If it passes the second check, the third check has a pass probability of 0.95. Draw a tree diagram and determine the overall probability that a machine passes all three checks.

27. Two bags contain balls. Bag 1 contains red and blue balls in a ratio such that the probability of drawing a red ball is 0.4; Bag 2 has a probability of drawing a red ball of 0.7. One bag is selected at random with equal probability, and then a ball is drawn from the selected bag. Draw a tree diagram and compute the overall probability that a red ball is drawn.

28. A medical test is represented using a tree diagram. The probability that a patient has a disease is 0.02. If a patient has the disease, the probability of a positive test is 0.95; if not, the probability of a positive test is 0.1. Construct the tree diagram and calculate the overall probability that a randomly selected patient tests positive.

29. A commuter has two routes to work. Route A is taken with probability 0.4, and Route B with probability 0.6. If taking Route A, the probability of arriving on time is 0.8; if taking Route B, the probability of arriving on time is 0.5. Draw a tree diagram and determine the overall probability of arriving on time.

30. A game is played in three rounds. The probability of winning any round is 0.3 and losing is 0.7 (assume independence). Draw a tree diagram for the three rounds and calculate the probability that the player wins exactly one round.