This worksheet focuses on using tree diagrams as a visual tool to represent sequences of events and calculate their probabilities. You will answer a series of questions ranging from simple exercises to challenging multi-stage scenarios. Work through each question carefully and show all your working.

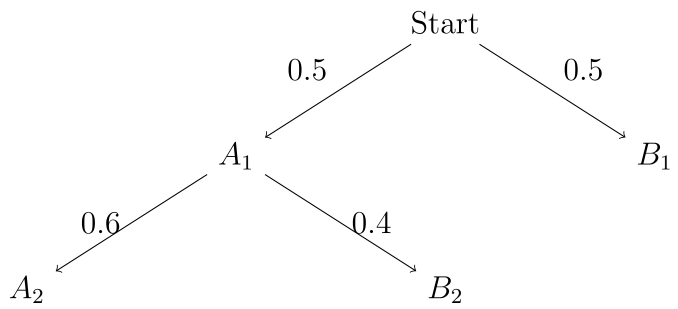
Easy Questions

1. Consider two independent tosses of a fair coin. Using a tree diagram in your head, calculate the probability of obtaining H on the first toss and T on the second toss.

2. A bag contains 1 red and 1 blue ball. You draw one ball, replace it and then draw again. Write down the probability of drawing a red ball then a blue ball using the multiplication rule.

3. In a two-stage experiment the first event has two possible outcomes, A and B, with $P(A) = 0.4$. Write the probability of B and explain why the total must equal 1.

4. Below is a tree diagram for a two-stage process. Use it to calculate the probability of the combined event A_1 then B_2.



5. In an experiment, the chance that event C occurs is 0.8, and if C occurs, the probability that event D occurs is 0.25. Calculate the probability that both C and D occur.

Intermediate Questions

6. A spinner has two outcomes on the first spin: X with probability 0.7 and Y with probability 0.3. If X occurs, a die is rolled and a 6 is obtained with probability 0.2. If Y occurs, the probability of rolling a 6 is 0.5. Use a tree diagram logic to calculate the overall probability of obtaining a 6.

7. A box contains 2 green and 3 yellow marbles. A marble is drawn, its colour recorded, and then the marble is replaced. This is followed by a coin toss that shows heads with probability 0.5. State the probability of drawing a green marble and then getting heads. Explain briefly how a tree diagram helps to organise these outcomes.

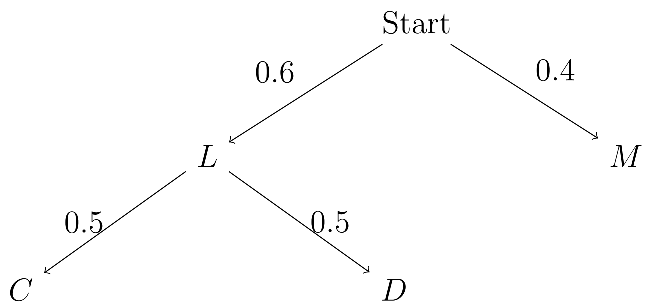
8. The following tree diagram has some missing probability values. The first branch splits into events M and N, and the second branch (from M) splits into P and Q. If $P(M) = 0.6$, $P(N) = 0.4$, and from M we have $P(P) = x$ and $P(Q) = 0.7$, find the value of x and explain your reasoning.

9. In a two-stage experiment, event R occurs on the first stage with probability 0.5. On the second stage, if R occurred, event S does not occur with probability 0.3. Determine the probability that R occurs followed by S not occurring.

10. Write a short explanation in plain text on how tree diagrams assist in organising multi-step probability problems.

11. A two-stage process has the following probabilities: First, event T occurs with probability $\frac{2}{3}$ and event U with probability $\frac{1}{3}$. If T occurs, event V occurs with probability $\frac{1}{4}$; if U occurs, event V occurs with probability $\frac{2}{5}$. Using these, compute the overall probability of event V.

12. Refer to the following tree diagram and find the probability of obtaining outcome D after starting from L.



13. In your own words, explain why a tree diagram is useful to represent sequential events compared to listing outcomes.

14. Given a tree diagram with branches (A, B) at the first stage and (C, D) at the second stage, state which branch represents the event A followed by D, and calculate its probability if $P(A) = 0.5$, $P(B) = 0.5$, and, when A occurs, $P(D) = 0.3$.

15. A bag contains 4 white and 6 black balls. A ball is drawn, its colour noted, and replaced. Then a coin is tossed (heads probability 0.5) and finally, a spinner with two outcomes, X and Y, with probabilities 0.3 and 0.7 respectively, is spun. Calculate the probability of drawing a white ball, getting heads and then outcome X.

16. An experiment consists of two stages. In the first stage, event E has probability 0.65 and event F has probability 0.35. If E occurs, then event G has probability 0.4, whereas if F occurs, event G occurs with probability 0.8. Compute the overall probability that G occurs.

17. In a two-stage process, event H occurs on the first stage with probability 0.55. Given that H occurs, event I occurs with probability 0.6. Write down the probability of H then I using a tree diagram approach.

18. A restaurant offers two types of meals: vegetarian (with probability 0.4) and non-vegetarian (with probability 0.6). After choosing a meal, a customer selects a dessert. If a vegetarian meal is chosen, the probability of selecting cake is 0.3, otherwise it is 0.5. On a piece of paper, draw a tree diagram for this scenario and calculate the probability of a non-vegetarian meal with cake for dessert.

19. In an experiment, event J does not occur with probability 0.2 in the first stage, and with probability 0.3 in the second stage (if needed). Using a tree diagram approach, calculate the probability that J does not occur in at least one of the two stages.

20. In a two-stage process, the first event can be either K (with probability 0.5) or L (with probability 0.5). If K occurs, the second event can be M (with probability 0.2) or N (with probability 0.8). If L occurs, the second event can be M (with probability 0.7) or N (with probability 0.3). Identify which branch represents K followed by N, and compute its overall probability.

Hard Questions

21. A game consists of three rounds. In round 1, a player can either win (probability 0.6) or lose (probability 0.4). In round 2, if the player won round 1, the chance to win is 0.5, otherwise it is 0.2. In round 3, regardless of previous outcomes, the probability of winning is 0.3. Construct a tree diagram on paper for all three rounds and calculate the probability of winning round 1, losing round 2, and winning round 3.

22. The following tree diagram represents a three-stage process. Using the diagram, calculate the probability of following the branch marked P then Q then R. Assume the probabilities are as shown:

$$P(\text{first branch } P) = 0.5,$$
$$P(\text{second branch } Q \mid P) = 0.4,$$
$$P(\text{third branch } R \mid Q) = 0.3.$$

Explain your computation.

23. A machine operates in three stages. The probability it works at stage 1 is 0.9. If it works at stage 1, its probability of working at stage 2 is 0.8, and if it works at stage 2, the probability of working at stage 3 is 0.85. Construct a tree diagram on paper to represent these events and compute the overall probability that the machine works through all three stages.

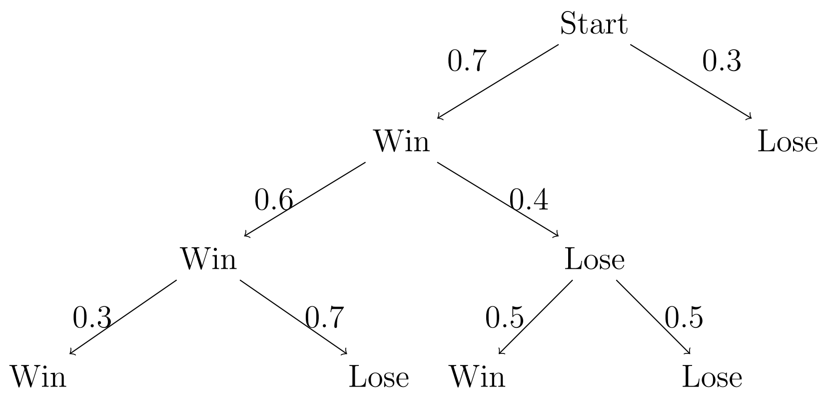
24. The following tree diagram (drawn below in `tikz`) represents a three-round quiz. Some branch probabilities are missing. It is known that:

$$P(\text{win in Round 1}) = 0.7, \quad P(\text{lose in Round 1}) = 0.3,$$

$$P(\text{win in Round 2} \mid \text{win in Round 1}) = 0.6, \quad P(\text{lose in Round 2} \mid \text{win in Round 1}) = 0.4,$$

$$P(\text{win in Round 3} \mid \text{lose in Round 2}) = 0.5.$$

Complete the diagram by indicating the missing branch probability for losing in Round 3 given a win in Round 2 (remember that the branches from a node sum to 1). Then, compute the probability of winning in Round 1, losing in Round 2, and then winning in Round 3.



25. A two-stage experiment has the following structure: In stage one, events A, B, and C occur with probabilities 0.3, 0.5, and 0.2 respectively. In stage two, each event splits into outcomes D and E. If the probability of D is 0.4 after A, 0.6 after B, and 0.5 after C, identify all the paths that result in outcome D, and calculate the total probability of D.

26. A lottery game works as follows. First, a ticket is drawn from a box containing 5 winning tickets and 15 losing tickets. If a winning ticket is drawn, a bonus round is played where a spinner lands on *Bonus* with probability 0.2. If a losing ticket is drawn, a consolation spinner is spun with a 0.4 chance of landing on *Bonus*. On a sheet of paper, draw a tree diagram to represent this process and compute the overall probability of getting a *Bonus*.

27. In an experiment, a bag contains 3 red and 7 blue tokens. A token is drawn without replacement. If a red token is drawn, a coin is tossed with heads probability 0.4; if a blue token is drawn, the coin shows heads with probability 0.8. Using a tree diagram approach (and noting the dependency due to no replacement), calculate the probability of drawing a red token and then getting heads. (Assume the bag is reset after the coin toss for simplicity of the tree.)

28. A two-stage process follows: In stage one, event F occurs with probability 0.5 and event G with probability 0.5. In stage two, if F occurs, event H occurs with probability 0.3, while if G occurs, event H occurs with probability 0.6. Using the tree diagram approach and the addition rule, determine the probability that event H occurs.

29. Given a tree diagram with two distinct branches leading to final outcomes: branch 1 with probability computed as 0.15, and branch 2 with probability 0.25, use the addition rule for disjoint events to calculate the probability that either branch 1 or branch 2 occurs.

30. A science experiment has three stages. In stage one, a researcher chooses one of two procedures: Procedure X with probability 0.4, or Procedure Y with probability 0.6. In stage two, under Procedure X, the experiment is successful with probability 0.5; under Procedure Y, it is successful with probability 0.7. In stage three, if the experiment was successful in stage two, an extra test is run that passes with probability 0.8. On paper, construct a complete tree diagram to represent these events and compute the overall probability of having Procedure Y, success in stage two, and passing the extra test.