ILLAWARRA TUTORING

This worksheet covers Permutations and Combinations. You will practise counting outcomes using permutations and combinations to solve complex probability problems. Work through the questions carefully and show all of your workings.

## Easy Questions

1. Determine the number of ways to arrange 3 distinct items in a row.
2. Calculate the number of ways to choose 2 items from 4 distinct items.
3. Please find the number of distinct arrangements of the letters in the word $(CAT)$.
4. Determine how many different 3-digit numbers can be formed using the digits (1, 2, 3) if no digit is repeated.
5. Compute 4!.

## Intermediate Questions

6. Determine in how many ways 5 distinct books can be arranged on a shelf.
7. Please find the number of ways to choose 3 representatives from a group of 10 candidates.
8. Calculate the number of distinct arrangements of the letters in the word $(LETTER)$. (Hint: Note that the letters $E$ and $T$ each appear twice.)
9. Determine the number of ways to select and arrange a president, vice-president and treasurer from 15 candidates.
10. Calculate the number of possible outcomes for the first, second and third places in a race of 8 runners.
11. Calculate the number of ways to choose 3 presenters from 7 people and then arrange them in a speaking order.
12. Determine the number of distinct arrangements of the letters in the word $(PROBABILITY)$, taking into account any repeated letters.
13. Calculate in how many ways 4 people can be seated in a row of 6 seats if exactly one seat must remain empty.

14. Determine the number of 5-letter words that can be formed using the alphabet, with no repetition of letters.

15. Compute the number of ways to award 3 prizes to 10 contestants if no contestant wins more than one prize.

16. Determine the number of ways to select a committee of 5 people from 12, and then choose one person from the committee as the president.

17. Calculate the number of ways to arrange 7 objects in a row such that two specific objects are not placed next to each other.

18. Determine the number of 4-digit even numbers that can be formed from the digits (1, 2, 3, 4, 5) without repetition. (Hint: Consider that the last digit must be even.)

19. Calculate the number of distinct necklaces that can be formed using 8 different coloured beads, assuming that rotations and reflections produce the same necklace.

20. Compute the number of ways to arrange 10 distinct flags on 3 flagpoles, where each flagpole must have at least one flag and the order on each flagpole matters. (Hint: First, arrange the flags in a row and then choose the positions for two dividers among the gaps between flags.)

# Hard Questions

21. Determine the number of ways to arrange 10 people in a row if two specific people must always sit together.

22. Compute the number of distinct arrangements of the letters in the word ($MISSISSIPPI$). (Hint: Count the frequency of each letter.)

23. Determine the number of ways to choose and arrange 5 books from 12 distinct books if the books must be arranged in alphabetical order (i.e. only the selection matters).

24. Calculate the number of possible 7-character passwords, formed from 62 possible characters (upper and lower case letters and digits), where each character is distinct and at least 2 characters are digits. (Hint: Compute the total number of arrangements then subtract those with fewer than 2 digits.)

25. Determine the number of distinct arrangements of the letters in the word ($ALGEBRA$) such that all vowels always appear together. (Hint: Treat the group of vowels as one unit and remember to account for repeated vowels.)

26. Compute the number of ways to form a committee of 7 people from 15 and then designate one member as the chairperson.

27. Determine the number of ways to distribute 8 distinct prizes among 4 students, ensuring that each student receives at least one prize. (Hint: Use the inclusion-exclusion principle.)

28. Calculate the number of seating arrangements for 10 people around a circular table, under the condition that two particular individuals must not sit next to each other. (Hint: Use the formula for circular permutations and subtract the arrangements where the pair is adjacent.)

29. Determine the number of ways to partition 10 distinct objects into 3 non-empty groups, where the order of the groups does not matter. (You may express your answer in terms of the Stirling numbers of the second kind.)

30. Determine the number of 8-character palindromic passwords that can be formed from a set of 36 characters (comprising 26 letters and 10 digits) when repetition is allowed. (Hint: Only the first 4 characters are free to choose.)