ILLAWARRA TUTORING

In this worksheet you will learn how to calculate the probability of an event given that another event has already occurred. Each question requires you to apply the definition of conditional probability, namely, $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)}$, to various situations.

Easy Questions

1. Answer the following: Write the formula for conditional probability.
2. A computer records $P(A) = \dfrac{1}{2}$ and $P(A \cap B) = \dfrac{1}{8}$. Calculate $P(B \mid A)$.
3. A factory reports that $P(R) = \dfrac{2}{5}$ for producing a red widget and $P(R \cap D) = \dfrac{1}{5}$ for producing a red defective widget. Find $P(D \mid R)$.
4. When tossing a fair coin twice, let event A be "Heads on the first toss" and event B be "Heads on the second toss". Calculate $P(B \mid A)$.
5. Given $P(A) = 0.4$ and $P(A \cap B) = 0.1$, compute $P(B \mid A)$.

Intermediate Questions

6. Suppose $P(A) = 0.7$ and $P(A \cap B) = 0.3$. Calculate $P(B \mid A)$.
7. Given $P(A) = 0.4$, $P(B) = 0.5$ and $P(A \cap B) = 0.2$, determine $P(B \mid A)$ and $P(A \mid B)$.
8. A jar contains 5 red balls and 7 green balls. One ball is drawn. If the drawn ball is red, what is the probability that a second ball drawn without replacement is green?
9. In a survey, 40% of respondents like tea and 25% like both tea and coffee. Find $P(\text{Coffee} \mid \text{Tea})$.
10. If 80% of emails are spam and 10% of spam emails contain the word "free", what is $P(\text{"free"} \mid \text{spam})$?
11. A company uses two machines. Machine X produces 60% of items with a 5% defect rate, and Machine Y produces 40% with a 10% defect rate. If an item is defective, calculate the probability it was produced by Machine Y.
12. An urn contains marbles numbered 1 to 10. If one marble is drawn at random, what is the probability that the number is even given that it is greater than 3?

13. In a school of 200 students, 120 study French, 80 study Spanish, and 50 study both. Calculate $P(\text{Spanish} \mid \text{French})$.

14. Suppose $P(\text{Rain}) = 0.3$ and $P(\text{Rain} \cap \text{Traffic Jam}) = 0.2$. Compute $P(\text{Traffic Jam} \mid \text{Rain})$.

15. A doctor administers a test that has a 90% chance of detection when a disease is present and a 20% chance of a false positive when absent. If 5% of the population has the disease, calculate the probability that a person who tests positive actually has the disease.

16. In an exam, 70% of students answer Question A correctly. Among those, 80% answer Question B correctly. Determine $P(B \mid A)$.

17. Let $P(A) = 0.5$, $P(B) = 0.4$, and $P(A \cap B) = 0.3$. Calculate $P(A \mid B)$ and comment on whether events A and B are independent.

18. Assume $P(A) = 0.6$, $P(B \mid A) = 0.5$, and $P(B \mid A') = 0.3$. Use the law of total probability to calculate $P(B)$.

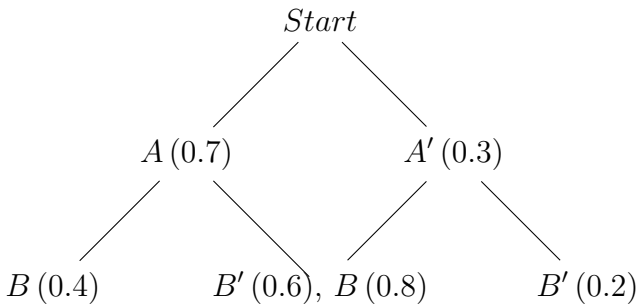
19. A disease test returns a positive result with probability 95% if a person has the disease and 5% if they do not. If 2% of the population are affected, determine the probability that a person does not have the disease given a negative test result.

20. An urn contains 2 white and 3 black balls. Two balls are drawn without replacement. Find the probability that the second ball is white given that the first ball drawn was black.

Hard Questions

21. In a factory, 40% of items are produced by Machine A and 60% by Machine B. Machine A has a defect rate of 2% and Machine B has a defect rate of 5%. If an item is found defective, compute the probability that it was produced by Machine A.

22. Use the following scenario to draw a probability tree diagram and then calculate $P(B)$ and $P(A \mid B)$: Event A occurs with probability 0.7 and its complement, A', with probability 0.3. If A occurs, B occurs with probability 0.4; if A' occurs, B occurs with probability 0.8.



23. A survey finds that 40% of people own a car, 30% own a motorcycle, and 15% own both. Find $P(\text{Motorcycle} \mid \text{Car})$.

24. Suppose $P(A) = 0.55$, $P(B) = 0.65$, and the probability that at least one occurs is 0.85. Compute $P(B \mid A)$.

25. In a card game, a single card is drawn from a standard deck. Let A be the event that the card is a heart and B be the event that the card is a face card. Determine $P(B \mid A)$.

26. A bank classifies its customers as low risk (80% of customers) or high risk (20%). The probability of default is 2% for low risk and 12% for high risk. If a customer defaults, compute the probability that they are from the high risk group.

27. In a two-stage exam, the probability of passing the first stage is 0.85. Given that a candidate passes the first stage, the probability of passing the second stage is 0.9. Compute the probability that a candidate fails the second stage given they passed the first stage.

28. A population is divided into groups X and Y in the proportions 0.6 and 0.4, respectively. The probability of exhibiting a certain trait in group X is 0.3, and in group Y it is 0.5. Given that a person exhibits the trait, calculate the probability that they belong to group X.

29. An urn contains 3 white, 5 red, and 2 blue balls. If one ball is drawn and is found to be red, what is the probability that a second ball drawn without replacement is blue?

30. Two events C and D occur with probabilities 0.65 and 0.55, respectively. However, if C occurs, the probability that D occurs increases to 0.75. Determine $P(D \mid C)$ and explain what this implies about the independence of C and D.