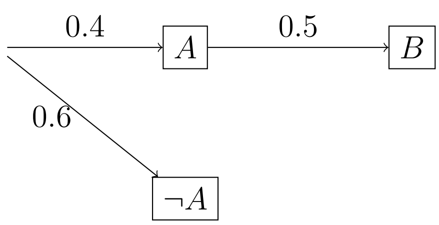
In this worksheet you will learn how to calculate the probability of an event given that another event has already occurred. You will practise using the definition of conditional probability and applying Bayes' theorem in several contexts. Work carefully through each question.

Easy Questions

1. The events A and B satisfy $P(B) = 0.5$ and $P(A \cap B) = 0.25$. Calculate the value of $P(A \mid B)$.
2. The events A and B have $P(B) = 0.6$, $P(A \cap B) = 0.3$ and $P(A) = 0.5$. Calculate $P(B \mid A)$.
3. Write the definition of conditional probability in your own words using the formula.
4. If an event B is certain (i.e. $P(B) = 1$), explain what the value of $P(A \mid B)$ is in terms of $P(A)$.
5. Determine whether the following statement is true or false: If events A and B are independent then $P(A \mid B) = P(A)$.

Intermediate Questions

6. A box contains balls where 60% are blue and 40% are red. A blue ball is correctly labelled as blue with probability 0.8, while a red ball is incorrectly labelled as blue with probability 0.3. A ball is drawn at random and is labelled blue. Compute the probability that the ball is actually blue.
7. Two fair coins are tossed. Given that at least one of the coins shows heads, compute the probability that both coins show heads.
8. A single card is drawn from a standard deck of 52 cards. If it is known that the card is red, what is the probability that it is a heart?
9. Using the probability tree diagram below, calculate $P(B \mid A)$ if the branch for A has probability 0.4, and the branch for B following A has probability 0.5.



(Hint: Use the definition of conditional probability.)

10. A medical test is 95% accurate for detecting a disease if a person has it, but it has a 5% false-positive rate for those without the disease. If 2% of the population has the disease, calculate the probability that a person has the disease given that they tested positive.

11. Two bags are available. Bag A contains 4 white and 6 black balls; Bag B contains 7 white and 3 black balls. One bag is chosen at random and a ball drawn from it is white. Find the probability that the ball came from Bag A.

12. Describe a real-life situation in which conditional probability is used and explain how it influences decision-making.

13. Given that $P(A) = 0.5$, $P(B) = 0.4$, and $P(A \cap B) = 0.2$, compute $P(A \mid B)$ and $P(B \mid A)$.

14. If $P(A \mid B) = 0.7$ and $P(B) = 0.5$, calculate $P(A \cap B)$.

15. In a survey, 60% of students like mathematics and 40% like physics. If 30% like both subjects, what is the probability that a student likes mathematics given that they like physics?

16. In a survey, 70% of respondents like tea, 50% like coffee, and 40% like both. Compute the probability that a respondent likes coffee given that they like tea.

17. Given that $P(A) = 0.5$, $P(B) = 0.6$, $P(A \mid B) = 0.8$, and $P(B \mid A) = 0.5$, calculate $P(A \cap B)$ and $P(B)$ as deduced from the provided conditional probabilities.

18. A process has two stages. The first event A occurs with probability 0.6. If A occurs, event B occurs with probability 0.3; if A does not occur, event B occurs with probability 0.5. On pen and paper, draw a probability tree diagram representing this scenario. Then, compute the overall probability of event B and the probability that A occurred given that B occurred.

19. In a game a die is rolled and a coin is tossed. Let A be the event that the die shows an even number and B be the event that the coin shows heads. If it is known that B has occurred, what is the probability that A also occurs?

20. An urn contains red, blue and green balls. The probability of drawing a red ball is 0.3 and a blue ball is 0.5. Given that a drawn ball is not green, compute the probability that it is blue.

Hard Questions

21. A university has 2000 students. Out of these, 800 study mathematics, 1200 study physics and 500 study both subjects. If a student is chosen at random and is known to study physics, calculate the probability that they also study mathematics.

22. The probability that it rains on a given day is 0.3. The probability of a traffic accident on a rainy day is 0.15; on a non-rainy day, it is 0.05. Compute the probability that it is raining given that an accident has occurred.

23. A machine produces parts in which 2% are defective. A quality control test correctly identifies a defective part 95% of the time, but also wrongly flags a non-defective part 10% of the time. Given that a part is flagged as defective, calculate the probability that it is actually defective.

24. In a survey, 70% of respondents like tea, 50% like coffee and 40% like both. If a respondent is known to like tea, what is the probability that they like coffee?

25. Suppose for events A and B it is given that $P(A \mid B) = 0.7$, $P(B) = 0.6$ and $P(A) = 0.5$. Verify whether these probabilities are consistent. If they are, compute $P(A \cap B)$; if not, explain the inconsistency.

26. At a certain school, 80% of students participate in at least one sport. Among all students, 50% play football, 40% play basketball and 20% play both sports. Calculate the probability that a student plays football given that they play basketball.

27. A virus test returns a positive result with probability 0.98 if a person is infected and 0.05 if not infected. With 1% infection prevalence in the population, compute the probability that a person is not infected given that they tested positive.

28. In a lottery two numbers are drawn. The probability that the first number is even is 0.5. If the first number is even, the probability that the second number is odd is 0.7; if the first number is odd, the probability that the second number is odd is 0.4. Calculate the probability that the first number is even given that the second number is odd.

29. A box contains 8 red, 12 blue and 10 green marbles. If a marble is drawn at random and it is known that the marble is not green, what is the probability that it is red?

30. A company classifies its employees into high-risk and not high-risk groups. Thirty percent of employees are high-risk, and for these employees the probability of a claim within a year is 0.2; for non-high-risk employees, the probability is 0.05. If an employee makes a claim, calculate the probability that they are in the high-risk group.