In this worksheet you will learn how to use the addition rule to combine probabilities for mutually exclusive and non–mutually exclusive events. You will solve problems involving simple applications as well as multi–step word problems using the addition rule:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Remember that if events are mutually exclusive then $P(A \text{ and } B) = 0$.

Easy Questions

1. Calculate $P(A \text{ or } B)$ given that $P(A) = 0.2$ and $P(B) = 0.35$ and the events are mutually exclusive.
2. Find $P(A \text{ or } B)$ where $P(A) = 0.5$, $P(B) = 0.3$, and A and B cannot occur together.
3. Define in your own words what it means for two events to be mutually exclusive.
4. Write an expression for $P(A \text{ or } B)$ using the addition rule when events A and B are mutually exclusive.
5. Explain why $P(A \text{ or } B)$ is not simply $P(A) + P(B)$ when events A and B are not mutually exclusive.

Intermediate Questions

6. Given $P(A) = 0.6$, $P(B) = 0.4$ and $P(A \text{ and } B) = 0.2$, calculate $P(A \text{ or } B)$.
7. A survey found that 65% of pupils like chocolate and 40% like vanilla ice cream. If 25% like both flavours, determine the probability that a pupil likes either chocolate or vanilla.
8. In a race, the probability that runner A finishes in the top 3 is 0.3 and runner B finishes in the top 3 is 0.25. If there is a 0.1 chance that both finish in the top 3, compute the probability that at least one of them finishes in the top 3.
9. If $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$ and $P(A \text{ and } B) = \frac{1}{12}$, compute $P(A \text{ or } B)$.
10. Calculate $P(A \text{ or } B)$ with $P(A) = 0.7$, $P(B) = 0.4$, and $P(A \text{ and } B) = 0.2$.
11. In a poll, 80% of respondents support policy X and 55% support policy Y. If 35% support both policies, find the percentage of respondents that support at least one policy.

12. Determine $P(A \text{ or } B)$ if $P(A) = 0.22$, $P(B) = 0.48$, and $P(A \text{ and } B) = 0.06$.

13. In a class, the probability that a student plays soccer is 0.5 and that he plays basketball is 0.4. If 15% play both sports, what is the probability that a student plays either sport?

14. From a deck of cards, the probability of drawing a heart is $\frac{1}{4}$ and the probability of drawing a king is $\frac{1}{13}$. Knowing there is one card that is both a heart and a king, calculate the probability that a drawn card is either a heart or a king.

15. For events A and B, if $P(A) = 0.55$, $P(B) = 0.45$, and $P(A \text{ and } B) = 0.25$, find $P(A \text{ or } B)$.

16. If the chance of raining on Saturday is 0.3 and on Sunday is 0.4, with a 0.1 chance that it rains on both days, what is the probability that it rains on at least one day?

17. In a survey, 40% of households have a pet cat and 30% have a pet dog, while 15% have both. Use the addition rule to find the probability that a household has either a pet cat or a pet dog.

18. In a university cohort, $P(\text{Study Maths}) = 0.65$ and $P(\text{Study Physics}) = 0.5$. If $P(\text{Study both}) = 0.35$, determine $P(\text{Study Maths or Physics})$.

19. Express $P(A \text{ or } B)$ in terms of $P(A)$, $P(B)$ and $P(A \text{ and } B)$. Then evaluate it when $P(A) = x$, $P(B) = y$, and $P(A \text{ and } B) = z$.

20. Using pen and paper, draw a Venn diagram for events A and B that overlap. Label the regions corresponding to only A, only B, and A and B. Then, write the addition rule formula for probabilities on your diagram.

Hard Questions

21. In a school, 20% of students do not play any sport while 75% play at least one of football or cricket. If 50% play football and 45% play cricket, determine the percentage of students that play both sports.

22. A medical study shows that $P(\text{Disease A}) = 0.1$ and $P(\text{Disease B}) = 0.08$ with an overlap of $P(\text{both diseases}) = 0.03$. Calculate the probability that a patient has at least one of the diseases.

23. Show that if $P(A) = \frac{n}{m}$, $P(B) = \frac{k}{m}$, and $P(A \text{ and } B) = \frac{p}{m}$, then $P(A \text{ or } B) = \frac{n+k-p}{m}$. Verify this formula for $n = 3$, $k = 4$, $p = 1$, and $m = 10$.

24. Consider a spinner divided into regions A and B such that $P(A) = 0.3$ and $P(B) = 0.3$ with an overlapping region where $P(A \text{ and } B) = 0.1$. If the spinner is spun once, find the probability that the arrow lands in either region A or B.

25. A box contains marbles that are classified as red and blue. The probability of drawing a red marble is 0.4, the probability of drawing a blue marble is 0.5, and the probability of drawing a marble that is both red and blue (striped) is 0.1. Find the probability of drawing a marble that is either red or blue.

26. Given that $P(A \text{ or } B) = 0.85$, $P(A) = 0.5$, and $P(A \text{ and } B) = 0.15$, solve for $P(B)$.

27. If two events are mutually exclusive with $P(A) = 0.35$ and $P(B) = 0.45$, calculate $P(A \text{ or } B)$. Explain your reasoning.

28. In a survey, 60% of voters support candidate X and 50% support candidate Y. If 30% support both candidates, what is the probability that a voter supports only one candidate?

29. In an experiment, event A occurs with probability 0.28 and event B occurs with probability 0.32. If they occur together with probability 0.12, find the probability that at least one of the events occurs.

30. Prove the general addition rule for any two events A and B, that is, show that

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

and provide a brief explanation of each step in your reasoning.