ILLAWARRA TUTORING

In this worksheet you will learn how to use the addition rule to combine probabilities for mutually exclusive and non–mutually exclusive events. You will practise recognising when to simply add probabilities and when you need to subtract the probability of the intersection to avoid double counting.

Easy Questions

1. For two mutually exclusive events where $P(A) = \frac{1}{4}$ and $P(B) = \frac{1}{3}$, calculate $P(A \text{ or } B)$.

2. A spinner is divided into three non-overlapping regions. If the probability that the arrow lands on red is $\frac{2}{5}$ and on blue is $\frac{1}{5}$, find the probability that the arrow lands on either red or blue.

3. Explain why for two mutually exclusive events, $P(A \text{ and } B)$ is 0. Provide a brief explanation in plain text.

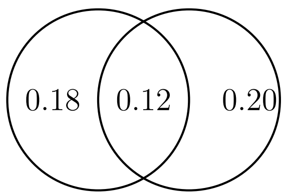
4. If $P(A) = 0.3$ and $P(B) = 0.4$, and events A and B cannot occur together, find $P(A \text{ or } B)$.

5. Given $P(A) = 0.5$, $P(B) = 0.6$, and $P(A \text{ and } B) = 0.2$, use the addition rule to calculate $P(A \text{ or } B)$.

Intermediate Questions

6. If $P(A) = 0.45$, $P(B) = 0.35$, and $P(A \text{ and } B) = 0.15$, compute $P(A \text{ or } B)$.

7. Below is a diagram representing events A and B. Using the diagram where the overlapping region has probability 0.12, the non-overlapping parts have probabilities 0.18 for A only and 0.20 for B only, calculate $P(A \text{ or } B)$.



8. A single card is drawn from a standard deck. If $P(\text{heart}) = \frac{1}{4}$ and $P(\text{king}) = \frac{1}{13}$, and knowing that the king of hearts is counted in both events, use the addition rule to find $P(\text{heart or king})$.

9. In a survey, 60% of respondents enjoy tea and 45% enjoy coffee. If 30% enjoy both, what is the probability (expressed as a percentage) that a randomly selected respondent enjoys either tea or coffee?

10. When rolling a fair die, let event A be obtaining an even number and event B be obtaining a number greater than 4. Compute $P(A \text{ or } B)$ using the addition rule.

11. In a school, 40% of students play basketball and 30% play soccer, with 10% playing both sports. Determine the probability that a randomly selected student plays at least one of these sports.

12. Given that $P(A) = 0.7$, $P(B) = 0.5$, and $P(A \text{ and } B) = 0.2$, calculate $P(A \text{ or } B)$.

13. In a lottery, the probability of winning a small prize is 0.05, the probability of winning a consolation prize is 0.1, and the probability of winning both is 0.02. Use the addition rule to find the probability of winning at least one prize.

14. In a class of 18 students, if 10 students like mathematics and 7 like science with 3 liking both, calculate the probability that a randomly selected student likes mathematics or science.

15. If from a relative frequency study $P(A) = 0.48$, $P(B) = 0.52$, and $P(A \text{ and } B) = 0.10$, find $P(A \text{ or } B)$.

16. If $P(A) = 0.3$, $P(B) = 0.4$, and $P(A \text{ and } B) = 0.12$, determine $P(A \text{ or } B)$.

17. The probability of an item being defective in Process A is 0.03 and in Process B is 0.05, with 0.01 being defective in both processes. Compute the probability that an item is defective in at least one process.

18. Demonstrate using a numerical example that if events A and B are mutually exclusive then the term $P(A \text{ and } B)$ is zero, and consequently $P(A \text{ or } B) = P(A) + P(B)$.

19. With $P(A) = 0.25$, $P(B) = 0.5$, and $P(A \text{ and } B) = 0.1$, show all your work to compute $P(A \text{ or } B)$.

20. Explain in your own words why subtracting $P(A \text{ and } B)$ is necessary when computing $P(A \text{ or } B)$ for events that are not mutually exclusive.

Hard Questions

21. Prove algebraically that for any two events A and B, the formula $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ avoids the double counting of the intersection. Support your proof with a numerical example.

22. In a school, 70% of students participate in music and 55% in sports, with 30% participating in both. Use the addition rule to calculate the probability that a student participates in at least one of these activities.

23. In a study, 60% of individuals use App A, 70% use App B, and 50% use both. Determine the probability that a randomly chosen individual uses at least one of the apps.

24. An urn contains balls that are either red, blue, or green. If $P(\text{red}) = 0.25$ and $P(\text{blue}) = 0.35$, but $P(\text{red or blue}) = 0.50$, calculate $P(\text{red and blue})$. Discuss why this result may be contradictory in a real-world context.

25. For events A and B with $P(A) = 0.6$ and $P(B) = 0.7$, use the addition rule to determine the possible range for $P(A \text{ or } B)$. Justify your answer based on the maximum possible intersection.

26. Given $P(A) = 0.8$ and $P(A \text{ and } B) = 0.2$, determine the range of possible values for $P(B)$ such that $P(A \text{ or } B) < 1$. Show all working.

27. Derive the addition rule for two events from first principles using set theory. In your answer explain each step and indicate clearly how the overlapping region is treated.

28. In an experiment, if $P(A) = 0.55$, $P(B) = 0.65$, and $P(A \text{ or } B) = 0.85$, determine $P(A \text{ and } B)$.

29. Given that $P(A \text{ or } B) = 0.9$ and $P(A) = 0.6$, determine the possible range for $P(B)$. Provide a justification for your answer using the addition rule and the constraints that probabilities must satisfy.

30. In an electoral poll, 48% of respondents support Party X and 39% support Party Y. If 15% support both parties, use the addition rule to determine the percentage of respondents who support at least one of the parties. Verify that the result is a valid probability.