In this worksheet you will learn to find the equations of tangents and normals to curves. You will practise computing derivatives (using basic rules and first principles only) and applying these to obtain the equations of lines that touch curves at a point. Remember that the tangent has the same gradient as the curve at the point of contact while the normal is perpendicular to the tangent.

Easy Questions

1. Find the equation of the tangent to $y = x^2$ at the point where $x = 1$.
2. Find the equation of the normal to $y = x^2$ at the point where $x = -2$.
3. Determine the equation of the tangent to $y = x^2$ at the point $(0, 0)$.
4. For the curve $y = 2x^2 + 1$, find the equation of the normal at the point where $x = 1$.
5. Show that the line $y = 3x - 5$ is tangent to the curve $y = 3x - 5$.

Intermediate Questions

6. Given $y = 3x^2 + 2x - 1$, compute the gradient of the tangent and write its equation at the point where $x = 1$.
7. For $y = 3x^2 + 2x - 1$, find the equation of the normal at $x = 1$.
8. For the curve $y = x^2 + 1$, let $(t, t^2 + 1)$ be a general point. Express the equation of the tangent at this point in terms of t.
9. Find the equation(s) of the tangent line(s) to $y = x^2$ that pass through the external point $(1, 3)$.
10. Determine the point(s) on $y = x^2$ where the tangent is perpendicular to the line $y = -x + 1$.
11. For $y = 2x^2 - 3x$, find the equation of the tangent at $x = 2$.
12. For $y = 2x^2 - 3x$, find the equation of the normal at $x = 2$.
13. Express the equation of the tangent to $y = x^2 + 2x$ at a general point $(a, a^2 + 2a)$ in terms of a.
14. Find the point(s) (a, a^2) on $y = x^2$ at which the tangent passes through the point $(3, 0)$.

15. For the curve $y = x^2 + 4$, determine the equation of the normal at the point where the tangent has a gradient of 4.

16. Find the equation of the tangent to the curve $y = 3 + x^2$ at $x = -1$.

17. Refer to the diagram below. Draw a clear diagram of the curve $y = x^2$ and its tangent at the point $(2, 4)$. Label the point of tangency and indicate the angle between the tangent and the positive x-axis.

18. For $y = 2x^2 - x + 3$, find the derivative at $x = 0$ and hence write the equation of the tangent at this point.

19. Determine the equation of the normal to $y = 5 - x^2$ at $x = 1$.

20. For the curve $y = x^2$, verify that the tangent at $(1, 1)$ touches the curve only at this point by finding the intersection of the tangent with the curve.

Hard Questions

21. Show that a line $y = mx + c$ is tangent to the curve $y = x^2 + k$ if and only if $c = k - \frac{m^2}{4}$.

22. Find the value of a so that the line $y = 2x + a$ is tangent to $y = x^2 - 4x + 6$.

23. Determine the point(s) on $y = x^2$ where the tangent is parallel to the line joining $(0, 0)$ and $(3, 5)$.

24. For the curve $y = 2x^2 + x + 1$, find the value of x where the tangent makes an angle of $45°$ with the positive x-axis.

25. For $y = x^2 - 3x + 2$, determine the equations of the tangent lines that intersect the y-axis at 5.

26. The line $y = 4x - 3$ is tangent to the curve $y = x^2$. Find the coordinates of the point of tangency.

27. A line passes through $(1, 2)$ and is tangent to $y = x^2$. Determine its equation.

28. For the curve $y = kx^2$, find the value of k such that the tangent at $x = 1$ is perpendicular to the line $y = 3x + 2$.

29. For the curve $y = -x^2 + 4x$, determine the coordinates of the point where the normal passes through the origin $(0, 0)$.

30. Determine the value of a such that the tangent at $x = a$ to the curve $y = 2x^2 - 5x + 3$ makes an angle of $60°$ with the positive x-axis.