ILLAWARRA TUTORING

In this worksheet you will learn to find the equations of tangents and normals to curves. You will practise determining gradients and using the point-gradient form to write equations of lines touching the curve at a given point. Remember that the normal is perpendicular to the tangent line.

Easy Questions

1. Consider the curve $y = x^2$. Find the gradient of the tangent at $x = 2$ and write down its equation.

2. Consider the line $y = 3x + 1$. State the gradient of the line and hence write the equation of the normal to the line at any point.

3. For the curve $y = x^2$, find the gradient of the tangent at $x = 1$ and hence determine the equation of the normal line at that point.

4. Given $y = 2x^2 - 3x + 1$, find the gradient of the tangent at $x = 1$ and hence write the equation of the tangent line.

5. For the curve $y = x^2$, determine the gradient of the normal at $x = -3$ and hence write its equation.

Intermediate Questions

6. Consider $y = x^2$. Find the gradient of the tangent at $x = 3$ and hence write down the equation of the tangent line at the point $(3, 9)$.

7. For the curve $y = x^2$, compute the gradient of the tangent at $x = -2$ and hence determine the equation of the normal line at the point $(-2, 4)$.

8. Given $y = x^2 + 2x$, find the gradient of the tangent at $x = 1$ and hence write the equations of both the tangent and the normal lines at the corresponding point.

9. Consider $y = x^2 + 4$. Determine the value of x for which the gradient of the tangent is 6, and then write the equation of the tangent line.

10. For the curve $y = x^2$, at what point is the tangent horizontal? Write the equation of the tangent and the normal at that point.

11. For $y = 2x^2 - 3x + 1$, find the gradient of the tangent at $x = 2$ and write the equations of both the tangent and the normal lines.

12. The curve $y = x^2 - 4x + 3$ has a tangent line parallel to $y = 2x + 5$. Determine the point(s) on the curve where this occurs.

13. Given $y = 3x^2 + 2$, find the gradient of the tangent and then determine the gradient of the normal line at $x = 1$. Write the corresponding equations.

14. For $y = -x^2 + 4x$, find the gradient of the tangent at $x = 1$ and hence write the equations of the tangent and normal lines at the corresponding point.

15. For the curve $y = x^2$, show that when the tangent is horizontal the corresponding normal line is vertical. Identify the point at which this occurs and write the equation of the normal line.

16. For $y = x^2 + 1$, find the equation of the normal line at the point $(1, 2)$.

17. For the curve $y = 4x^2$, determine the value of x for which the tangent is parallel to the line $y = 8x + 3$ and write the corresponding tangent equation.

18. Find the equation of the tangent line to $y = x^2 + 3$ at $x = -1$ and hence determine the equation of the normal line.

19. The curve $y = x^2 - 2x + 1$ is a perfect square. Determine the coordinates of its vertex and then write the equation of the normal line at that point.

20. For $y = 2x^2 - x$, find the gradient of the tangent at $x = 0$ and hence write the equations for both the tangent and the normal lines at the corresponding point.

Hard Questions

21. Given $y = x^2$, determine all points on the curve where the tangent line makes an angle of 45° with the positive x-axis.

22. For the curve $y = x^2 - 4x + 5$, find the equations of the tangent lines that are perpendicular to $y = 2x + 1$.

23. Consider the curve $y = x^2 + x$. Find the point of tangency such that the tangent line passes through $(2, 6)$ and write its equation.

24. Given $y = 3x^2 - 4x + 2$, determine the coordinates at which the tangent line is parallel to $y = x + 1$ and then write the equation of the normal line at that point.

25. A line normal to the curve $y = x^2 + 3x + 2$ passes through $(1, 0)$. Determine the point of tangency on the curve.

26. For $y = 2x^2 - 7x + 3$, first find the point(s) where the curve intersects the x-axis. Then write the equations of the tangent and the normal lines at the relevant point.

27. For the curve $y = x^2 - 6x + 8$, find the point at which the normal is parallel to the line $y = -x + 4$ and then determine the equation of this normal line.

28. For the curve $y = 2x^2 - 3x + 1$, let a tangent at $x = a$ intercept the x-axis at $(p, 0)$. Express p in terms of a and find the value(s) of a for which $p = a$.

29. The tangents to the curve $y = x^2$ at $x = 0$ and $x = 3$ meet at a point. Determine the coordinates of their intersection.

30. For the curve $y = 4x^2 - 12x + 9$, find the tangent and the normal at $x = 2$. Then verify that the gradients of these lines multiply to -1.