ILLAWARRA TUTORING

This worksheet will help you understand and calculate the second derivative to analyse the concavity of functions and identify points of inflection. Work through each question carefully and show all your working.

Easy Questions

1. Calculate $f'(x)$ and $f''(x)$ for the function $f(x) = x^2$. Then, state whether the graph is concave up or concave down.
2. Find the first derivative and second derivative of $f(x) = 3x + 4x^2$. Explain what the second derivative tells you about the curvature of the graph.
3. For $f(x) = x^3$, compute $f'(x)$ and $f''(x)$. Identify any point of inflection and state the intervals where the function is concave up or down.
4. Find $f''(x)$ for $f(x) = 5 - 2x^2$ and describe the concavity of the graph.
5. For $f(x) = x^4$, determine $f'(x)$ and $f''(x)$. Discuss the concavity of the function and state whether there is a point of inflection.

Intermediate Questions

6. For $f(x) = x^3 - 3x$, calculate $f'(x)$ and $f''(x)$. Identify the point of inflection and the intervals on which the function is concave up or concave down.
7. Determine $f''(x)$ for $f(x) = x^4 - 4x^2$. Find the values of x where the concavity may change and state the intervals of concavity along with the inflection points.
8. Compute the first and second derivatives of $f(x) = 2x^3 + x^2 - x$. Find the value of x at which the function has an inflection point and describe the concavity on either side.
9. Given $f(x) = -x^3 + 6x^2 - 9x + 1$, find $f'(x)$ and $f''(x)$. Determine the concavity of the function and identify the point of inflection.
10. For $f(x) = x^3 + 3x^2 + 3x + 1$, calculate $f''(x)$ and hence determine the point where the concavity changes.
11. Find $f'(x)$ and $f''(x)$ for $f(x) = x^3 - 6x^2 + 9x + 1$. Identify the inflection point and state on which intervals the function is concave up or down.
12. Compute the second derivative of $f(x) = x^4 - 8x^2 + 16$. Determine the values of x where $f''(x) = 0$ and analyse the concavity of the function.

13. For $f(x) = -2x^3 + 9x^2 - 12x + 5$, determine $f''(x)$ and find the point of inflection. Discuss the concavity on each interval divided by this point.

14. Given $f(x) = \frac{1}{3}x^3 - x^2$, calculate $f''(x)$ and find the inflection point. Describe how the sign of $f''(x)$ changes around that point.

15. Evaluate the second derivative of $f(x) = 2x^2 - 4x + 1$ and state whether the function is concave up or down.

16. For $f(x) = 4x^4 - 2x^3 + x^2$, determine $f'(x)$ and $f''(x)$. Simplify your answer and discuss any potential change in concavity.

17. Determine the intervals of concavity for $f(x) = -x^4 + 4x^3$ by finding its first and second derivatives. Identify the critical points where $f''(x) = 0$.

18. For $f(x) = x^3 - 9x$, compute the second derivative and specify the intervals in which the function is concave up.

19. Compute $f'(x)$ and $f''(x)$ for $f(x) = x^4 - 2x^2$. Determine the values for which $f''(x) = 0$ and discuss how these relate to inflection points.

20. Analyse the function $f(x) = 3x^3 - 12x + 7$ by computing $f'(x)$ and $f''(x)$. Find the inflection point and discuss the concavity on either side of this point.

Hard Questions

21. Let $f(x) = x^5 - 5x^3 + 4x$. Compute $f'(x)$ and $f''(x)$. Determine the exact values of x where $f''(x) = 0$ and use a sign chart to identify the intervals of concavity and the inflection points.

22. Consider the function $f(x) = x^4 - 4x^3 + 6x^2 - 4x + 1$. Calculate $f''(x)$ and explain why the function has no true point of inflection even if $f''(x) = 0$ at a point.

23. Given $f(x) = \frac{1}{3}x^3 - x$, compute the first and second derivatives. Identify the inflection point and justify your answer by testing the sign of $f''(x)$ on either side of the inflection point.

24. For $f(x) = \frac{1}{4}x^4 - x^2$, compute $f'(x)$ and $f''(x)$. Find the critical values where $f''(x) = 0$ and determine the intervals of concavity.

25. Consider $f(x) = 2x^3 - 9x^2 + 12x + 1$. Determine the first and second derivatives, then find the exact coordinate(s) of any inflection point by solving $f''(x) = 0$.

26. Let $f(x) = x^5 - 10x^3 + 9x$. Find $f'(x)$ and $f''(x)$. Factorise $f''(x)$ completely and determine all values of x where the concavity of $f(x)$ changes.

27. Given $f(x) = x^4 - 8x^2 + 16$, compute its second derivative and determine the exact values of x for which $f''(x) = 0$. Discuss how these values relate to the concavity of the function.

28. For $f(x) = x^5 + 2x^3 - 8x$, calculate $f'(x)$ and $f''(x)$. Factorise $f''(x)$, identify its zeros, and describe the intervals of concavity of the function.

29. Consider $f(x) = x^4 - 2x^3 - 3x^2 + 4x$. Find the first and second derivatives, then solve $f''(x) = 0$ to identify the inflection points. Provide the exact values for these points.

30. Let $f(x) = x^3 - 6x^2 + 9x + e$, where e is a constant. Compute $f'(x)$ and $f''(x)$. Explain why the constant e does not influence the concavity of the function.