ILLAWARRA TUTORING

This worksheet will help you understand and calculate the second derivative to analyse the concavity of functions and to identify points of inflection. Work through each question carefully and show all your working.

Easy Questions

1. Compute the second derivative of $f(x) = x^2$.
2. For $f(x) = x^2 + 3x + 1$, calculate the second derivative and state whether the function is concave up or down.
3. Given $f(x) = x^3$, find $f''(x)$ and evaluate it at $x = 2$.
4. Determine the second derivative of $f(x) = 2x^2 - 5x + 4$ and explain why the function does or does not have an inflection point.
5. For $f(x) = 4x - 7$, find the second derivative and comment on the concavity.

Intermediate Questions

6. For $f(x) = x^3$, compute $f''(x)$ and state the intervals where the function is concave up and concave down.
7. For $f(x) = 3x^2 - 5x + 2$, calculate $f''(x)$ and indicate the concavity of the function over all real numbers.
8. Given $f(x) = x^4$, compute $f''(x)$ and discuss the concavity of the function.
9. For $f(x) = 2x^3 - 3x^2 - 12x + 5$, determine $f''(x)$ and find any point of inflection.
10. For $f(x) = x^3 + 3x^2$, calculate $f''(x)$, and identify the point where concavity changes.
11. For $f(x) = \frac{1}{3}x^3 - x$, find $f''(x)$ and state the intervals of concavity.
12. Given $f(x) = x^2 - 4x + 7$, compute $f''(x)$ and determine whether the graph is concave up or down.
13. For $f(x) = x^3 - 3x$, calculate $f''(x)$ and identify any inflection points.
14. Given $f(x) = -2x^3 + 6x$, compute $f''(x)$ and find the coordinate(s) of the inflection point.

15. For $f(x) = -x^4 + 4x^2$, determine $f''(x)$, and solve for the values of x where the concavity may change.

16. For $f(x) = 2x^4 - 8x^2$, compute $f''(x)$ and determine the inflection points by solving $f''(x) = 0$.

17. Given $f(x) = (x - 1)^4$, find $f''(x)$ and discuss the concavity of the function.

18. For $f(x) = -x^2$, calculate $f''(x)$ and state the nature of the concavity.

19. For $f(x) = 0.5x^3 + x^2$, compute $f''(x)$ and identify any inflection point.

20. Given $f(x) = x^3 + x$, calculate $f''(x)$ and determine the interval(s) of concavity as well as any inflection points.

Hard Questions

21. Explain in detail why a point where $f''(x) = 0$ may not be a point of inflection. Provide an example of a function for which $f''(x) = 0$ at a certain point but no change in concavity occurs.

22. For $f(x) = x^4 - 2x^3 - 12x^2 + 5$, compute $f''(x)$ and determine all possible inflection points, justifying your answer with sign changes of $f''(x)$.

23. Given $f(x) = 2x^3 - 9x^2 + 12x + 1$, find $f''(x)$, and then specify the intervals on which the function is concave up and concave down. Indicate any point of inflection.

24. Let $f(x) = ax^3 + bx^2 + cx + d$, where $a \neq 0$. Show that $f''(x) = 6ax + 2b$, and explain how the coefficients a and b determine the location of the unique zero of $f''(x)$ and, hence, the change in concavity.

25. For $f(x) = x^3 - 6x^2 + 9x + 7$, compute $f''(x)$ and determine the concavity of the function. Identify the point of inflection and verify your result using a sign diagram for $f''(x)$.

26. Given $g(x) = -x^4 + 4x^3 + x^2 - 10x + 3$, find $g''(x)$ and determine the intervals on which $g(x)$ is concave up or concave down. Clearly show your reasoning.

27. Prove that if a function f is twice differentiable and $f''(x) = 0$ for all x, then $f(x)$ must be a linear function. Present your proof in clear steps.

28. For $f(x) = x^3 + kx$, find all values of k such that $f(x)$ has an inflection point at $x = 0$. Justify your answer.

29. Given $h(x) = x^4 - 4x^3 + 6x^2$, compute $h''(x)$ and determine all values of x at which the concavity changes. Explain the method used to determine these points.

30. For $f(x) = -3x^4 + 12x^3 - 9x^2 + 2$, first compute $f''(x)$ and then solve the inequality $f''(x) > 0$. Use your solution to specify the intervals on which the function is concave up.