This worksheet explores how to calculate both average and instantaneous rates of change, linking calculus to real world situations. You will investigate these concepts using algebraic methods and real world models.

Easy Questions

1. Calculate the average rate of change of $f(x) = 3x + 2$ between $x = 1$ and $x = 5$. Provide your answer as a single number.

2. For the function $f(x) = x^2$, determine the average rate of change between $x = 2$ and $x = 4$. Write your answer in simplest form.

3. A car travels along a straight road and its distance (in kilometres) is given by $s(t) = 60t$ where t is in hours. Find the average rate of change of $s(t)$ between $t = 1$ and $t = 3$. Explain what this rate represents.

4. Explain in your own words what is meant by the instantaneous rate of change of a function at a specific point.

5. For $f(x) = x^2$, compute the average rate of change between $x = 3$ and $x = 3.1$ and discuss how it approximates the instantaneous rate of change at $x = 3$.

Intermediate Questions

6. Determine the average rate of change of $f(x) = x^2 + 1$ between $x = 1$ and $x = 4$. Show all your working.

7. The function f has the following values: $f(2) = 4$, $f(3) = 9$, $f(4) = 16$. Calculate the average rate of change between $x = 2$ and $x = 4$, and state what this value represents.

8. Draw the graph of $f(x) = x^2$, draw the secant line between $x = 1$ and $x = 3$, and then sketch the tangent line at $x = 2$. Explain the significance of the slopes of these lines.

9. A town's population is modelled by $P(t) = 1000 + 50t + 2t^2$, where t is the time in years since 2000. Calculate the average rate of change of the population between $t = 3$ and $t = 7$, and state what this tells you about the town's growth.

10. An object moves such that its distance from a starting point is given by $s(t) = 4t^2 + 2t$ (with s in metres and t in seconds). Find the average speed between $t = 1$ and $t = 3$.

11. Find the average rate of change of $f(x) = x^3$ between $x = -1$ and $x = 2$. Simplify your answer.

12. For the function $f(x) = \sqrt{x}$, determine the average rate of change between $x = 4$ and $x = 9$. Show your method.

13. Compute the average rate of change of $f(x) = 2x^2 + 3$ between $x = 2$ and $x = 5$. Explain the impact of the constant term on your calculation.

14. A container's volume (in litres) when water is added over time is given by $V(t) = t^2 + 10$, where t is in minutes. Determine the average rate of change of volume between $t = 3$ and $t = 7$, and interpret its meaning.

15. Write the difference quotient for a function $f(x)$ over the interval from x to $x + h$. Explain how this quotient approximates the instantaneous rate of change as h becomes very small.

16. For the function $f(x) = x^2$, approximate the instantaneous rate of change at $x = 3$ by computing the average rate of change from $x = 3$ to $x = 3.001$. Discuss your result.

17. Calculate the average rate of change of $f(x) = \frac{1}{x}$ between $x = 2$ and $x = 4$. Express your answer in a simplified form.

18. For $f(x) = -x^2 + 4$, find the average rate of change between $x = 0$ and $x = 2$. Explain the significance of the negative sign in your answer.

19. Given $f(x) = x^2 - 2x$, first calculate the average rate of change between $x = 3$ and $x = 3.1$, then discuss how this approximates the instantaneous rate of change at $x = 3$.

20. In an economic model, the price of a commodity is given by $P(q) = 50 - 0.5q$, where q represents the quantity produced. Determine the average rate of change of $P(q)$ as q increases from 20 to 40, and provide a brief explanation of its economic interpretation.

Hard Questions

21. Using the definition of the derivative, find the instantaneous rate of change of $f(x) = (x + 1)^2$ at $x = 3$. (Hint: Write out the difference quotient and simplify.)

22. The graph of $f(x) = x^2$ is provided. Using the diagram, estimate the instantaneous rate of change at $x = 2$ and compare it with the average rate of change between $x = 1.5$ and $x = 2.5$. Explain any differences you observe.

23. A company's cost (in dollars) to produce x items is given by $C(x) = 0.05x^2 + 2x + 100$. Find the instantaneous rate of change of cost when $x = 50$ by approximating it using the average rate of change over a short interval. State any assumptions made.

24. Derive a general expression for the average rate of change of $f(x) = x^2$ over the interval $[a, a + h]$. Express your answer in terms of a and h, and simplify.

25. Show that for any linear function $f(x) = mx+b$, the average rate of change between any two points is always equal to m, the instantaneous rate of change. Provide a clear algebraic explanation.

26. Use the limit definition of the derivative to show that the instantaneous rate of change of $f(x) = 3x + 2$ is 3. Provide each step of your reasoning.

27. A ball is thrown vertically upward so that its height (in metres) at time t (in seconds) is given by $h(t) = -5t^2 + 20t + 2$. Estimate the instantaneous velocity (rate of change of height) at $t = 2$ by calculating the average rate of change over a very short interval around $t = 2$. Explain your method.

28. In an economic context, if the profit function is given by $P(x) = -x^2 + 10x - 16$, calculate the average rate of change between production levels $x = 2$ and $x = 6$. Interpret what this rate of change might imply about marginal profit.

29. For $f(x) = \sqrt{x+4}$, compute the average rate of change between $x = 5$ and $x = 9$. Show all steps involved in your computation.

30. Rainfall accumulation (in millimetres) is modelled by $R(t) = t^2 + 2t$, where t represents time in hours. Determine the instantaneous rate of change in rainfall at $t = 3$ by using a short interval approximation. Then, explain in a brief paragraph what this instantaneous rate tells us about the rainfall at that moment.