In this worksheet you will develop your skills in differentiating quotients of functions by applying the quotient rule. Recall that if $f(x) = \dfrac{g(x)}{h(x)}$ then

$$f'(x) = \frac{g'(x)h(x) - g(x)h'(x)}{[h(x)]^2}.$$

Work through all questions and show full working.

Easy Questions

1. Differentiate $f(x) = \dfrac{2x+3}{5}$.
2. Differentiate $f(x) = \dfrac{5}{x}$.
3. Differentiate $f(x) = \dfrac{x^2}{x}$.
4. Differentiate $f(x) = \dfrac{x}{x^2+1}$.
5. Differentiate $f(x) = \dfrac{x+1}{x-1}$.

Intermediate Questions

6. Differentiate $f(x) = \dfrac{3x^2+2}{x+4}$.
7. Differentiate $f(x) = \dfrac{x^3+2x}{x^2+1}$.
8. Differentiate $f(x) = \dfrac{5x^2-4x+1}{2x-3}$.
9. Differentiate $f(x) = \dfrac{x^2+x}{x+2}$.
10. Differentiate $f(x) = \dfrac{4x-7}{x^2}$.
11. Differentiate $f(x) = \dfrac{x^3-2x+1}{x-1}$.

12. Differentiate $f(x) = \dfrac{x^2 + 6}{3x - 2}$.

13. Differentiate $f(x) = \dfrac{2x^2 - 3}{x + 5}$.

14. Differentiate $f(x) = \dfrac{x^2 - 9}{x - 3}$.

15. Differentiate $f(x) = \dfrac{x^2 + 3x + 2}{x + 1}$.

16. Differentiate $f(x) = \dfrac{2x^2 + 5x - 3}{x - 4}$.

17. Differentiate $f(x) = \dfrac{x^2 - 4}{x^2 + 4}$.

18. Differentiate $f(x) = \dfrac{3x + 1}{x^2}$.

19. Differentiate $f(x) = \dfrac{2x + 3}{x - 2}$.

20. Differentiate $f(x) = \dfrac{x^3 + x}{x + 1}$.

Hard Questions

21. Differentiate $f(x) = \dfrac{x^3 + 2x^2 - x + 3}{x^2 - 1}$. Simplify your answer.

22. Differentiate $f(x) = \dfrac{x^4 - 2x^2 + 1}{x^3 + 1}$. Simplify your result.

23. Differentiate $f(x) = \dfrac{2x^3 + 3x^2 - x + 1}{x^2 + 2x + 1}$. Simplify your answer.

24. Differentiate $f(x) = \dfrac{x^3 - x}{x^2 + x - 2}$. Show all steps.

25. Differentiate $f(x) = \dfrac{3x^3 + 4x^2 - 5x + 2}{2x^2 - x - 1}$. Simplify your final answer.

26. Differentiate $f(x) = \dfrac{x^4 + x^3 - x + 1}{x^2 + 1}$. Simplify the derivative.

27. Differentiate $f(x) = \dfrac{5x^3 - 3x^2 + 2}{x^3 - 2x + 1}$. Express your answer in a simplified form.

28. Differentiate $f(x) = \dfrac{x^3 + x^2}{x^2 - x}$. Simplify your result.

29. Differentiate $f(x) = \dfrac{2x^3 + x - 4}{x^2 + 3x + 2}$. Simplify your answer.

30. Differentiate $f(x) = \dfrac{x^4 - 16}{x^2 - 4}$. Show all steps and simplify.