This worksheet focuses on developing your skills in differentiating quotients of functions by applying the quotient rule. You will work through easy, intermediate, and challenging problems designed to reinforce and deepen your understanding of the quotient rule. Remember that the quotient rule states that if $f(x) = \dfrac{g(x)}{h(x)}$, then

$$f'(x) = \frac{h(x)g'(x) - g(x)h'(x)}{[\,h(x)]^2}.$$

Read each question carefully and show all your working.

Easy Questions

1. Differentiate $f(x) = \dfrac{x^2}{x}$.
2. Differentiate $f(x) = \dfrac{x+1}{x-1}$.
3. Differentiate $f(x) = \dfrac{2x}{x+3}$.
4. Differentiate $f(x) = \dfrac{x^2+1}{x}$.
5. Differentiate $f(x) = \dfrac{3x}{2x}$.

Intermediate Questions

6. Differentiate $f(x) = \dfrac{x^3+2x}{x-1}$.
7. Differentiate $f(x) = \dfrac{3x^2-x+5}{2x+3}$.
8. Differentiate $f(x) = \dfrac{4x^3+6}{x^2-1}$.

9. Differentiate $f(x) = \dfrac{x^2 - 4}{x + 2}$.

10. Differentiate $f(x) = \dfrac{\sqrt{x}}{x + 1}$, rewriting $\sqrt{x}$ as $x^{\frac{1}{2}}$.

11. Differentiate $f(x) = \dfrac{x^2 + 3}{3x^2 - 2}$.

12. Differentiate $f(x) = \dfrac{5x - 1}{x^2 + 1}$.

13. Differentiate $f(x) = \dfrac{x^3 - 2}{2x^2 + x}$.

14. Differentiate $f(x) = \dfrac{2x^2 - 3x + 1}{x - 2}$.

15. Differentiate $f(x) = \dfrac{x + 3}{x^2}$.

16. Differentiate $f(x) = \dfrac{2x^3}{x^2 + 4}$.

17. Differentiate $f(x) = \dfrac{3x^2 + 2x + 1}{x + 5}$.

18. Differentiate $f(x) = \dfrac{2x^2 + x}{x^2 - 4}$.

19. Differentiate $f(x) = \dfrac{x^2 + 1}{2x - 3}$.

20. Differentiate $f(x) = \dfrac{4x - 5}{x^2 + 2}$.

Hard Questions

21. Differentiate $f(x) = \dfrac{x^3 - 2x + 1}{x^2 - x - 2}$.

22. Differentiate $f(x) = \dfrac{2x^4 - 3x^2 + 1}{x^3 + x - 1}$.

23. Differentiate $f(x) = \dfrac{x^4 + 2x^3 - x + 4}{2x^3 - x^2 + 3}$.

24. Differentiate $f(x) = \dfrac{3x^3 + 5x^2 - x + 7}{x^3 - 2x + 1}$.

25. Differentiate $f(x) = \dfrac{x^3 + x^2 + x + 1}{x^2 - x + 2}$.

26. Differentiate $f(x) = \dfrac{5x^2 - 2x + 3}{3x^3 + x}$.

27. Differentiate $f(x) = \dfrac{x^2 + 4x + 3}{x^2 - 4}$.

28. Differentiate $f(x) = \dfrac{x^3 - x}{x^2 + x + 1}$.

29. Differentiate $f(x) = \dfrac{2x^2 - 5x + 3}{x^2 + 3x + 2}$.

30. Differentiate $f(x) = \dfrac{x^4 - x^2 + 1}{2x^3 - 3x + 4}$.