ILLAWARRA TUTORING

This worksheet focuses on the product rule. You will learn how to differentiate products of functions by applying the product rule formula. Follow the instructions on each question and show all your working.

Easy Questions

1. Differentiate $f(x) = (x)(x^2)$ using the product rule.
   Hint: Let $u = x$ and $v = x^2$. Then $f'(x) = u'v + uv'$.

2. Differentiate $f(x) = (2x)(x + 3)$ using the product rule.
   Hint: Identify $u = 2x$ and $v = x + 3$.

3. Write down the product rule formula for differentiation and explain briefly what each part represents.

4. Differentiate $f(x) = (3)(x^3)$ using the product rule.
   Hint: Remember that the derivative of a constant is zero.

5. Differentiate $f(x) = (x + 1)(x - 1)$ using the product rule.
   Hint: Identify the two factors as $u = x + 1$ and $v = x - 1$.

Intermediate Questions

6. Differentiate $f(x) = (x^2)(x + 2)$ using the product rule.
   Hint: Let $u = x^2$ and $v = x + 2$.

7. Differentiate $f(x) = (x + 1)(x - 3)$ using the product rule.

8. Differentiate $f(x) = (2x - 1)(x^2 + 4)$ using the product rule.

9. Differentiate $f(x) = (x^3)(2x^2 - 5)$ using the product rule and simplify your answer.

10. Differentiate $f(x) = (3x + 2)(x^2 - x + 1)$ using the product rule.

11. Differentiate $f(x) = (\tan x)(x^2)$ using the product rule.
    Hint: Remember that $\dfrac{d}{dx}(\tan x) = \sec^2 x$.

12. Differentiate $f(x) = (x^2)(\ln x)$ using the product rule.

13. Differentiate $f(x) = (x^3 + 1)(2x^2 - 3x + 4)$ using the product rule and simplify your result.

14. Differentiate $f(x) = x(x^2 + 2x + 1)$ using the product rule.

15. Differentiate $f(x) = (\cos x)(x^2)$ using the product rule.
Hint: $\dfrac{d}{dx}(\cos x) = -\sin x$.

16. Differentiate $f(x) = (x - \sin x)(x + \cos x)$ using the product rule.
Hint: Compute the derivatives $u' = \frac{d}{dx}(x - \sin x)$ and $v' = \frac{d}{dx}(x + \cos x)$.

17. Differentiate $f(x) = (e^x)(x^2)$ using the product rule.

18. Differentiate $f(x) = (4x - 3)(x^3 + 2)$ using the product rule and simplify your answer.

19. Differentiate $f(x) = (5x^2)(3x - 7)$ using the product rule.

20. Differentiate $f(x) = (x^2 + 2)(x^3 - 1)$ using the product rule and simplify the result.

Hard Questions

21. Differentiate $f(x) = (x^2 + 2x)(x^3 - 3x + 5)$ using the product rule and simplify your answer as much as possible.

22. Differentiate $f(x) = (3x^3 - 2x + 4)(x^4 + 5x^2)$ using the product rule and then calculate $f'(1)$.

23. Differentiate $f(x) = (x - 1)(x^2 + 2x + 3)(x + 4)$ by first grouping the last two factors as one function and then applying the product rule twice.

24. Show that the derivative of $f(x) = (x^2)(x^3)$ is $5x^4$ by applying the product rule.

25. Differentiate $f(x) = (x^3 - x)(2x^2 + 3)$ using the product rule and factorise your final answer.

26. Differentiate $f(x) = (\ln x)(x - 1)$ using the product rule and simplify your answer.

27. Differentiate $f(x) = (e^x)(x^2 + x)$ using the product rule and then compute $f'(0)$.

28. Differentiate $f(x) = (\cos x)(\sin x)$ using the product rule and simplify your answer using a trigonometric identity.

29. Differentiate $f(x) = (x^2)(\cos x)(\sin x)$ by treating it as a product of two functions where one is (x^2) and the other is $(\cos x)(\sin x)$. Then use the product rule appropriately and simplify your answer.

30. Differentiate $f(x) = (2x^2 - 3)(3x^2 + 4x - 5)$ using the product rule. Then, set $f'(x) = 0$ and solve for x (you may leave your answer in a factorised or unsimplified form if necessary).