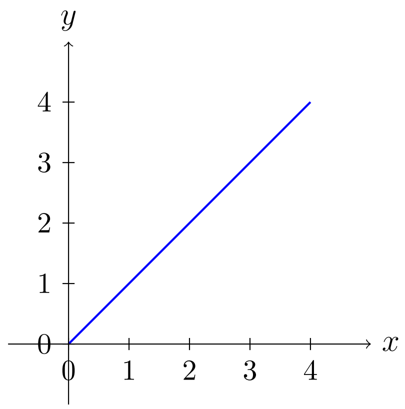
ILLAWARRA TUTORING

In this worksheet you will develop an understanding of the idea of a gradient at a specific point on a curve and how it represents the rate of change. You will work on conceptual and numerical tasks that focus on calculating and interpreting gradients through secant and tangent analyses.

Easy Questions

1. Write in one sentence what is meant by the gradient of a curve at a point.
2. Find the gradient of the line passing through the points (2, 3) and (5, 7).
3. Determine the gradient of the line with equation $y = 3x - 2$.
4. Examine the diagram below and state the gradient of the line drawn.

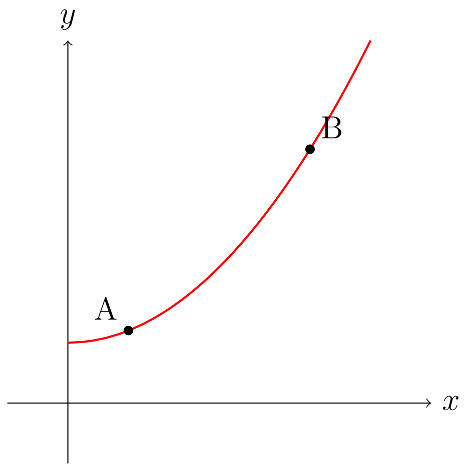


5. Explain why the gradient of the curve $y = x$ is 1 at every point.

Intermediate Questions

6. Explain how the gradient of a secant line between two points on a curve represents the average rate of change over that interval.
7. For the function $f(x) = x^2$, find the average gradient between $x = 1$ and $x = 3$.
8. For the function $f(x) = x^2$, compute the average gradient between $x = 2$ and $x = 2.5$.
9. For the function $f(x) = x^2$, explain in words how you might approximate the gradient at $x = 2$ using the average gradients of nearby secant lines.

10. Below is a diagram of a curve with two distinct points. Draw and label a secant line joining these points.



11. Determine the gradient of the function $f(x) = 2x + 3$ between any two distinct points on its graph.

12. If the gradient of a curve at a point is 5, what does this indicate about the behaviour of the curve at that point?

13. A curve passes through (1, 4) and (3, 8). Calculate the gradient of the secant line joining these points.

14. A curve has a tangent that is horizontal at a certain point. Explain what the gradient is at that point.

15. In a diagram of a rising curve, if one tangent line is visibly steeper than another, compare their gradients.

16. Consider a function whose tangent at $x = 1$ has a gradient of 3. Explain what might happen to the gradient if you move to a point where the curve appears flatter.

17. Explain, in your own words, the difference between the average gradient over an interval and the instantaneous gradient at a point.

18. Describe a real-world situation where the gradient (as a rate of change) is an important measure and explain why.

19. If the tangent at a point on a curve makes an angle θ with the positive x-axis, express the gradient in terms of θ.

20. Determine the gradient of a line that makes an angle of 45° with the positive x-axis.

Hard Questions

21. For the function $f(x) = x^2$, discuss how the slopes of secant lines drawn between $(x, f(x))$ and a nearby point approach the gradient at x.

22. Consider a continuous curve with varying steepness. Explain how you might identify the point where the curve is ascending most steeply.

23. A curve has a horizontal tangent at a point, yet the average gradient measured over an interval containing that point is positive. Discuss possible shapes or behaviour of the curve around that point.

24. A tangent to a curve makes an acute angle with the x-axis. Explain how the numerical value of the gradient reflects the measure of this angle.

25. If a curve has a tangent at a point that is parallel to the line $y = 3x + 1$, what is the gradient of the curve at that point?

26. For a smooth and continuous curve, discuss why the gradient at any point is a reliable representation of the instantaneous rate of change.

27. Two different curves intersect at a point and have the same tangent at that point. What does this imply about the gradients of the curves at the intersection?

28. Explain how the gradient at a point on a curve may be used to approximate the change in the function's value for a small change in x.

29. A section of road is represented by a smooth curve, and the gradient at a certain point is measured to be 7. Discuss what this gradient indicates in practical terms regarding the steepness of the road and any potential implications for safety and design.