In this worksheet you will learn about the gradient of a curve. You will explore how the gradient at a specific point represents the instantaneous rate of change and how it can be determined or approximated using graphical and numerical methods.

Easy Questions

1. Write down the gradient of the line given by $y = 3x + 2$.

2. In one sentence, explain what the gradient of a line represents.

3. Determine the gradient of the line $y = -5x + 7$.

4. Write the equation of a straight line with gradient 4 and y-intercept -3.

5. Calculate the gradient of the line passing through the points $(1, 2)$ and $(4, 8)$ using the formula for gradient.

Intermediate Questions

6. Describe in a few sentences what is meant by the tangent to a curve at a point.

7. The following table represents values of x and y for a certain smooth curve:

x	y
2	5
3	8
4	13

   Explain how you would approximate the gradient of the curve between $x = 2$ and $x = 4$.

8. In your own words, explain how the gradient at a point on a curve represents the rate of change.

9. A graph shows the distance travelled (y) versus time (x). Write a short explanation of what the gradient of this graph represents.

10. Given two points on a line, $A(2, 3)$ and $B(6, 11)$, use the rise over run method to compute the gradient.

11. Consider the diagram below. Using the provided figure, calculate the gradient by identifying the vertical (rise) and horizontal (run) components.
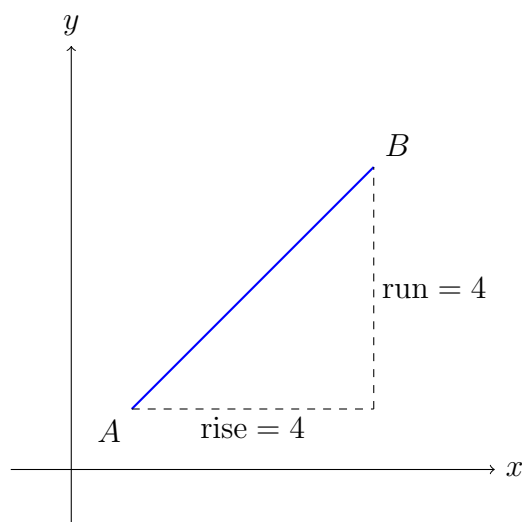
12. On a smooth curve, two close points P and Q have coordinates approximately $(3, 7)$ and $(3.5, 8.5)$ respectively. Explain how you can use these points to approximate the gradient at P.

13. The values of a function near $x = 2$ are given by $f(1.9) = 4.1$ and $f(2.1) = 4.9$. Calculate an approximate gradient of the curve at $x = 2$.

14. Describe the difference between a secant line and a tangent line on a curve.

15. In a short paragraph, discuss how the gradient influences the steepness and direction (increasing or decreasing) of a curve.

16. A table gives the temperature T (in degrees Celsius) recorded at different times t (in hours):

t	T
1	15
3	19
5	23

Describe how you would use these data to estimate the rate at which the temperature is rising.

17. Explain what it means if the gradient of a distance-time graph is increasing as time increases.

18. Write a short explanation about the significance of positive and negative gradients on any curve.

19. Explain what it means for a curve at a point if the gradient is zero.

Hard Questions

20. Prove that in any linear function of the form $y = mx + c$, the gradient is constant and equal to m.

21. Consider the curve $y = \dfrac{1}{x}$. Explain, without using formal differentiation techniques, why the steepness of the curve changes as x increases or decreases.

22. A cubic curve is approximately represented by the points $D(1, 2)$, $E(1.1, 2.5)$ and $F(1.2, 3.1)$. Explain how you would estimate the gradient at $x = 1.1$ using these points.

23. The gradient of a line is related to the angle of inclination θ by the relation $m = \tan\theta$. If a line makes an angle of $30°$ with the horizontal, calculate its gradient.

24. Prove that if two non-vertical lines are perpendicular then the product of their gradients is -1.

25. A ramp has a gradient calculated as $\dfrac{3}{4}$. Explain how to compute the gradient as a percentage grade and determine its value.

26. A road is made up of two connected segments. The first segment rises from $(0, 0)$ to $(5, 10)$ and the second from $(5, 10)$ to $(10, 12)$. Estimate the overall gradient from the start of the road to the end.

27. A smooth curve is divided into three sections with estimated gradients of 2, 0 and -3 respectively. Discuss briefly how these differing gradients affect the appearance of the curve.

28. On a blank sheet of paper, sketch a smooth curve of your choice. At two different points, draw the tangent lines and use a ruler to estimate the rise and run. Then, calculate the approximate gradients at those points. Write a few sentences describing any differences observed between the two gradients.