In this worksheet you will explore the concept of the gradient of a curve. You will learn what the gradient at a specific point represents in terms of rate of change, how to calculate average gradients using pairs of points, and how to interpret gradient values in different contexts.

Easy Questions

1. Write a short definition of the gradient of a curve. In your answer include a discussion of what it represents in terms of rate of change.

2. Calculate the gradient between the points $(2, 3)$ and $(5, 11)$. Show your working.

3. For a straight line given by the equation $y = 4x + 1$, state its gradient and explain how you know.

4. Find the average gradient between the points $(1, 2)$ and $(4, 14)$ by calculating $\frac{\textit{change in y}}{\textit{change in x}}$.

5. In a brief paragraph, explain what a positive gradient, a negative gradient, and a zero gradient reveal about the behaviour of a curve.

Intermediate Questions

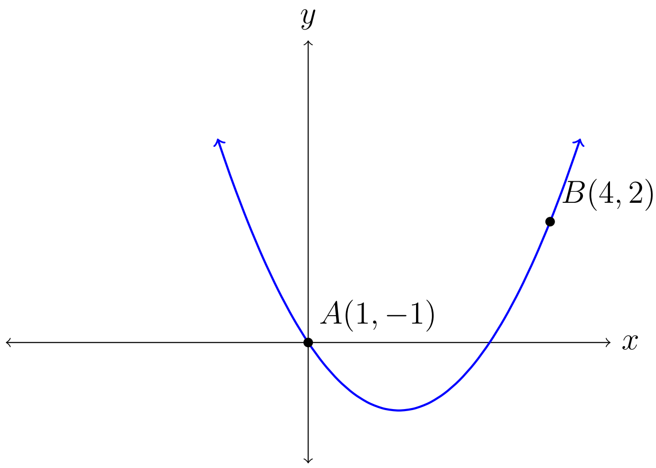
6. The table below shows values of x and y from a curve:

x	y
1	2
3	8
5	18

Calculate the average gradient between $x = 1$ and $x = 5$.

7. Explain in your own words how the gradient of a curve describes its steepness and direction of change.

8. Refer to the diagram below. Two points A and B lie on a smooth curve. Using the coordinates given at $A(1, -1)$ and $B(4, 2)$, calculate the gradient of the secant line joining these points.



9. A curve passes through the points $(2, 5)$ and $(6, 17)$. Compute the average gradient between these two points.

10. For the straight line $y = 2x - 3$, verify that the gradient is the same between the points $(1, -1)$ and $(4, 5)$ as it is between $(3, 3)$ and $(7, 11)$.

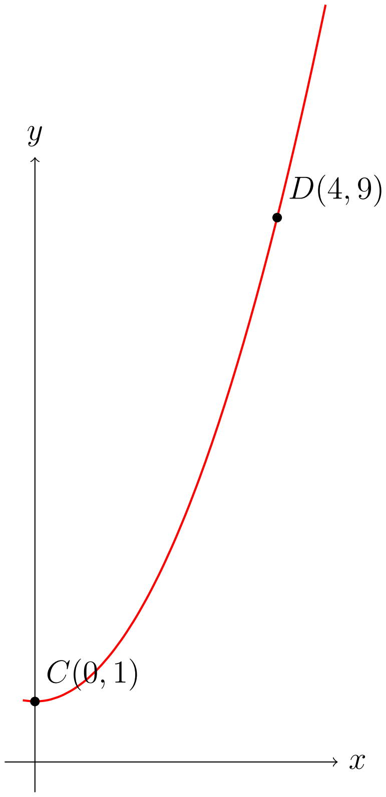
11. The curve is defined by the rule $y = x^2$. Calculate the average gradient between $x = 2$ and $x = 4$.

12. Complete the following sentence: The gradient of a line is equal to the ratio of (*change in y*) to (*change in x*); that is, $m = \frac{---}{---}$.

13. A road has a steady incline represented by a straight line on a distance versus elevation graph. If the elevation increases by 5 metres for every 100 metres of horizontal distance, explain the meaning of its gradient.

14. The coordinates of a curve near a point are given as follows: $(2, 5)$ and $(2.5, 7.5)$. Estimate the gradient between these points.

15. Look at the graph drawn below. Points C and D on the curve have coordinates $C(0, 1)$ and $D(4, 9)$. Calculate the average gradient between C and D.



16. Explain what is meant by a steep gradient and describe how it would appear on a graph.

17. Discuss how the gradient of a curve relates to the rate at which the quantity y changes with respect to x in a real-world situation.

18. Two different curves have average gradients 3 and -2 respectively. Compare the meaning of these gradients.

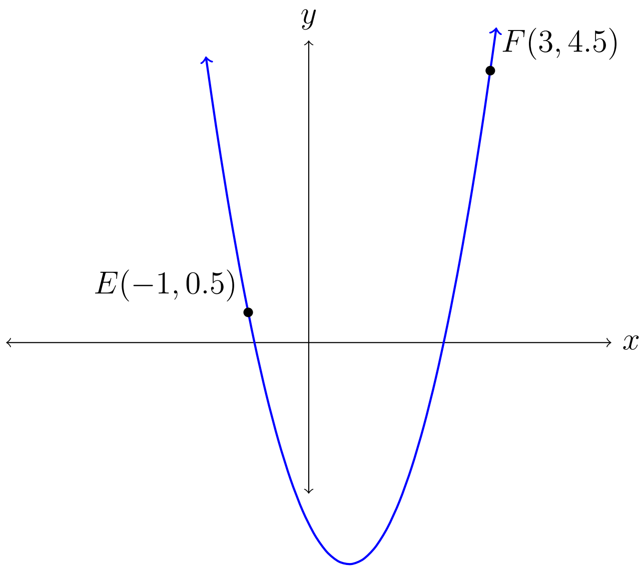
19. A curve is sampled over two intervals: from $x = 1$ to $x = 3$ and from $x = 3$ to $x = 5$. If the calculated average gradients are 2 and 4 respectively, discuss what this suggests about the behaviour of the curve.

20. Complete the following statement: If a curve has an average gradient of a, then for each unit increase in x, the value of y increases by approximately a units. Write your answer in the blank.

Hard Questions

21. The function $y = x^2$ is given. Use the points $(2.9, 8.41)$ and $(3.1, 9.61)$ to approximate the gradient of the curve when $x = 3$. Show your calculations.

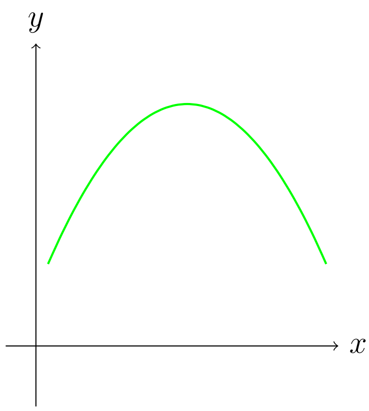
22. The diagram below shows a smooth, non-linear curve. Using the points marked $E(-1, 0.5)$ and $F(3, 4.5)$ on the curve, estimate the gradient of the secant between these points.



23. Explain why the gradient of a vertical line is undefined.

24. Calculate the gradient of the line joining $(-2, 5)$ and $(3, -1)$. Then provide a brief interpretation of your result.

25. The following diagram shows a segment of a curve with a horizontal section. Explain what a zero gradient signifies, and identify the point(s) at which this occurs on the diagram.



26. A pipeline follows a gradual slope represented by a curve. At point P the horizontal distance is 50 metres and the elevation is 10 metres, while at point Q the horizontal distance is 80 metres and the elevation is 22 metres. Calculate the average gradient between points P and Q and explain its meaning.

27. The table below shows values of x and y for a curve:

x	y
0	0
2	3
4	9
6	12

Determine which interval $[x_1, x_2]$ has the steepest average gradient and calculate that gradient.

28. Discuss how computing average gradients over smaller and smaller intervals can lead to the concept of the instantaneous rate of change. Write a brief explanation.

29. A hill's elevation (in metres) for given distances (in kilometres) is recorded below:

Distance	*Elevation*
0.0	150
0.5	165
1.0	180
1.5	200

Estimate the gradient of the hill between 0.5 km and 1.0 km, and then estimate the gradient around 1.25 km by comparing adjacent intervals.

30. Write a short paragraph that explains what is meant by the gradient at a specific point on a curve. In your explanation, discuss its importance for understanding the rate of change and how it is useful in real-life applications.