ILLAWARRA TUTORING

In this worksheet you will develop a deep understanding of differentiation by deriving the derivative of basic functions using its definition. You will work through problems applying the limit process to find the derivative from first principles.

Easy Questions

1. Please use the definition of the derivative to show that for $f(x) = 7$, the derivative $f'(x) = 0$.
2. Please use the first principles definition of the derivative to prove that for $f(x) = x$, the derivative is $f'(x) = 1$.
3. Derive the derivative of $f(x) = 2x$ using the limit definition. Explain each step in your working.
4. Using differentiation from first principles, show that for $f(x) = x^2$, we have $f'(x) = 2x$.
5. Use the definition of the derivative to find $f'(x)$ if $f(x) = x + 4$. Write your answer with clear justification.

Intermediate Questions

6. Use first principles to derive the derivative of $f(x) = 3x^2$. Show all substitution and limit steps.
7. Derive the derivative of $f(x) = x^3$ using the definition of the derivative. Simplify your answer.
8. Using differentiation from first principles, find $f'(x)$ for $f(x) = 4x^2$.
9. Derive the derivative of $f(x) = \sqrt{x}$ using the first principles approach. Hint: Rationalise the numerator.
10. Establish from first principles that if $f(x) = \dfrac{1}{x}$ then $f'(x) = -\dfrac{1}{x^2}$. Use common denominator techniques where necessary.
11. Show by first principles that $f(x) = \dfrac{1}{x^2}$ has derivative $f'(x) = -\dfrac{2}{x^3}$.
12. Use the definition of the derivative to determine $f'(x)$ for $f(x) = 4x^2 + 2x + 1$. Explain every step.

13. Derive the derivative of $f(x) = x^2 - 3x$ from first principles. Write a clear explanation.

14. Using differentiation from first principles, show that for $f(x) = -x^2$, the derivative is $f'(x) = -2x$.

15. Find the derivative of $f(x) = x^4$ using first principles. Include all algebraic manipulation.

16. Derive $f'(x)$ for $f(x) = \sqrt{x+1}$ using the definition of the derivative. Hint: Write the function as $(x+1)^{\frac{1}{2}}$.

17. Use the limit definition of the derivative to find $f'(x)$ for $f(x) = x^2 + x$. Provide a step-by-step explanation.

18. Determine $f'(x)$ for $f(x) = 2x^3 + 5x$ from first principles. Show all working clearly.

19. Show that using the definition of the derivative, for $f(x) = x^2 - 4$, the derivative is $f'(x) = 2x$.

20. Using differentiation from first principles, derive the derivative of $f(x) = 5x^2 - 3x + 7$. Explain your reasoning.

Hard Questions

21. Prove from first principles that for $f(x) = x^5$, the derivative is $f'(x) = 5x^4$. Include a full expansion of the binomial expression.

22. Provide a detailed derivation of the derivative of $f(x) = \sqrt{x}$ using the rationalisation technique in the difference quotient.

23. Use the definition of the derivative to derive $f'(x)$ for $f(x) = \dfrac{1}{x+3}$. Clearly show how you obtain a common denominator to simplify the difference quotient.

24. Establish from first principles that for $f(x) = \dfrac{1}{x^2+1}$, the derivative is given by an expression in x. Show all algebraic steps including combining fractions.

25. Derive the derivative of $f(x) = \dfrac{x^3 - x}{x}$ using first principles. Begin by simplifying the function algebraically and then apply the limit process.

26. Derive the derivative of $f(x) = (x+2)^2$ from first principles. Your solution should begin by expanding the square and then applying the limit definition.

27. Using the definition of the derivative, find $f'(x)$ for $f(x) = \dfrac{3}{x-1}$. Hint: Rewrite the function as $3(x-1)^{-1}$ and simplify the difference quotient.

28. Consider the piecewise function

$$f(x) = \begin{cases} x^2, & x \leq 1, \\ 2x, & x > 1. \end{cases}$$

Use first principles to determine the left-hand and right-hand derivatives at $x = 1$. Explain whether $f'(1)$ exists.

29. Using first principles, show that the derivative of $f(x) = |x|$ is given by $f'(x) = \dfrac{x}{|x|}$ for $x \neq 0$. Explain why the derivative does not exist at $x = 0$.

30. As a challenge, use the definition of the derivative to find $f'(x)$ for $f(x) = x^{\frac{1}{3}}$. Discuss the nature of the derivative at $x = 0$.