ILLAWARRA TUTORING

In this worksheet you will develop a deep understanding of differentiation by deriving the derivative of basic functions using its definition. All questions must be answered using the limit definition of the derivative (first principles). Do not use any shortcut rules or methods developed in later units.

Easy Questions

1. Use the definition of the derivative from first principles to find the derivative of
$$f(x) = 5.$$

2. Using the limit definition, derive the derivative of
$$f(x) = x.$$

3. Find the derivative from first principles for the function
$$f(x) = 2x + 3.$$

4. Derive the derivative of
$$f(x) = x^2$$
using the definition of the derivative.

5. In your own words, explain what the limit
$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
represents in the context of the derivative.

Intermediate Questions

6. Use differentiation from first principles to find the derivative of
$$f(x) = 3x^2.$$

7. Derive the derivative of
$$f(x) = x^3$$
by using the definition of the derivative.

8. Find the derivative of
$$f(x) = \frac{1}{x}$$
for
$$x \neq 0$$
using first principles.

9. Derive the derivative of
$$f(x) = \sqrt{x}$$
for
$$x > 0$$
using the limit definition.

10. Using differentiation from first principles, compute the derivative of
$$f(x) = 2x^2 + 3x + 4.$$

11. Find the derivative of
$$f(x) = x^2 - 4x + 7$$
by applying the limit definition.

12. Derive the derivative of
$$f(x) = 4x^2$$
from first principles.

13. Using first principles, compute the derivative of
$$f(x) = 5x^2 - 3x + 2.$$

14. Derive the derivative from first principles for
$$f(x) = 2x^3.$$

15. Use the definition of the derivative to find the derivative of
$$f(x) = -x^2 + x.$$

16. Find the derivative of
$$f(x) = x^3 + x$$
using differentiation from first principles.

17. Derive the derivative of
$$f(x) = x^4$$
through the limit process.

18. Using first principles, show that the derivative of the constant function
$$f(x) = 3$$
is zero.

19. Prove that for
$$f(x) = x^2$$
, the derivative obtained from the limit definition is
$$2x.$$
Detail each step.

20. Find the derivative of
$$f(x) = (x+2)^2$$
by first expanding the expression and then using the definition of the derivative.

Hard Questions

21. Use the limit definition of the derivative to find the derivative of
$$f(x) = \frac{1}{x+1}$$
for
$$x \neq -1.$$

22. Derive the derivative of
$$f(x) = \sqrt{x+3}$$
using differentiation from first principles.

23. Compute the derivative of
$$f(x) = (2x+1)^2$$
from first principles. Show all algebraic steps.

24. Derive the derivative of
$$f(x) = \frac{1}{1+x^2}$$
using the definition of the derivative. Simplify your final answer.

25. Using first principles, find the derivative of
$$f(x) = \frac{1}{\sqrt{x}}$$
for
$$x > 0.$$

26. Explain in detail how the limit definition of the derivative provides the instantaneous rate of change of a function. Use a specific example of your choice to illustrate your explanation.

27. Given
$$f(x) = x^3$$
, use the first principles approach to compute
$$f'(2).$$

28. Show that the derivative of
$$f(x) = x^2$$
at any point
$$x$$
is
$$2x$$
by carefully using the limit definition. Provide justification for each algebraic manipulation.

29. Use the limit definition to find the derivative of
$$f(x) = \frac{1}{x^2}$$
for
$$x \neq 0.$$

30. Consider the function
$$f(x) = \sqrt{2x+3}.$$
Derive its derivative from first principles and simplify your answer completely.