ILLAWARRA
TUTORING

This worksheet develops a deep understanding of differentiation from first principles. In this worksheet, you will derive the derivative of basic functions using the definition of the derivative.

Easy Questions

1. Use the definition of the derivative to find the derivative of $f(x) = x$.

2. Use first principles to find the derivative of $f(x) = 7$.

3. Use differentiation from first principles to compute the derivative of $f(x) = -x$.

4. Using the limit definition of the derivative, determine the derivative of $f(x) = x + 3$.

5. Use first principles to show that the derivative of $f(x) = 0$ is zero.

Intermediate Questions

6. Derive the derivative of $f(x) = x^2$ using first principles.

7. Using first principles, find the derivative of $f(x) = 3x^2$.

8. Derive the derivative of $f(x) = x^3$ from first principles.

9. Use the limit process to find the derivative of $f(x) = ax^2 + bx + c$, where a, b, and c are constants.

10. Find the derivative of $f(x) = 4x^2 - 5x + 2$ using the definition of the derivative.

11. Use first principles to derive the derivative of $f(x) = \dfrac{1}{x}$.

12. Derive the derivative of $f(x) = \sqrt{x}$ by applying the limit definition and rationalising the numerator.

13. Using first principles, determine the derivative of $f(x) = \dfrac{1}{\sqrt{x}}$.

14. Find the derivative of $f(x) = 2x^3 + 5$ using the first principles approach.

15. Verify from first principles that the derivative of $f(x) = x^4$ is $4x^3$.

16. Use the limit definition to show that the derivative of $f(x) = 1 - x^2$ is $-2x$.

17. Using first principles, calculate the derivative of $f(x) = 3x^2 - 2x + 7$.

18. Find the derivative of $f(x) = -2x^2 + x$ by applying the definition of the derivative.

19. Use first principles to show that the derivative of $f(x) = ax + b$ is a, where a and b are constants.

20. Derive the derivative of $f(x) = 5x^2$ using the limit process.

Hard Questions

20. Prove using first principles that $f(x) = x^4$ has a derivative of $4x^3$. Provide each step of your binomial expansion.

21. Using first principles, derive the derivative of $f(x) = \dfrac{1}{x}$ by rewriting it as x^{-1} and simplifying the difference quotient.

22. For the function $f(x) = x^{\frac{3}{2}}$, use differentiation from first principles to find its derivative.

23. Show using first principles that the derivative of $f(x) = |x|$ does not exist at $x = 0$. Explain your reasoning.

24. Derive the derivative of $f(x) = \sqrt{x}$ from first principles, clearly showing how you rationalise the numerator.

25. Using first principles, determine the derivative of $f(x) = \dfrac{1}{\sqrt{x}}$ by expressing the function in exponential notation.

26. Calculate the derivative of $f(x) = (x - 1)^2$ using the definition of the derivative. Expand the expression before simplifying.

27. Derive the derivative of $f(x) = (2x + 3)^2$ using first principles. Begin by expanding the squared term.

28. Using the definition of the derivative, find the derivative of $f(x) = (x + 1)^3$. Show all your steps including the binomial expansion.

29. Given $f(x) = ax^2 + bx + c$, use first principles to derive the expression for $f'(x)$ and then evaluate $f'(a)$ in terms of a and b.