ILLAWARRA TUTORING

This worksheet focuses on the topic of Derivatives and Indices. In this unit, you will learn to differentiate functions involving indices, extending your ability to handle exponential expressions. Apply the power rule to functions with integer, fractional, and negative exponents. Show all your working and simplify your answers where possible.

## Easy Questions

1. Differentiate $f(x) = x^4$.
2. Differentiate $f(x) = x^{\frac{1}{3}}$.
3. Differentiate $f(x) = 3x^{-2}$.
4. Differentiate $f(x) = 4x^{\frac{1}{3}}$.
5. Differentiate $f(x) = -5x^5$.

## Intermediate Questions

6. Differentiate $f(x) = 7x^{\frac{5}{2}}$.
7. Differentiate $f(x) = -4x^{-4} + 3x^{\frac{2}{3}}$.
8. Differentiate $f(x) = \dfrac{3}{4}x^7 - 5x^{\frac{3}{2}}$.
9. Differentiate $f(x) = 8x^{\frac{1}{2}} + 10x^3 - x^{\frac{4}{3}}$.
10. Differentiate $f(x) = 2x^{\frac{3}{4}} - 6x^{-\frac{1}{4}}$.
11. Differentiate $f(x) = 5x^{\frac{3}{2}} + 3x^{\frac{1}{2}} - 9x^{\frac{5}{2}}$.
12. Differentiate $f(x) = \dfrac{2}{3}x^{\frac{11}{2}} - 8x^3$.
13. Differentiate $f(x) = -2x^{\frac{5}{3}} + 7x^{\frac{8}{3}}$.
14. Differentiate $f(x) = 3x^{\frac{7}{2}} - 4x^{-3} + 2x^{\frac{1}{2}}$.
15. Differentiate $f(x) = 8x^{-\frac{3}{2}} + 2x^{\frac{5}{2}}$.
16. Differentiate $f(x) = x^3 + 5x^{\frac{2}{3}} - 4x^{-\frac{5}{3}}$.

17. Differentiate $f(x) = -4x^8 + 3x^{\frac{7}{2}} - 2x^{\frac{5}{2}}$.

18. Differentiate $f(x) = 10x^{\frac{1}{3}} - 6x^{\frac{4}{3}} + 2x^{\frac{7}{3}}$.

19. Differentiate $f(x) = 7x^{\frac{13}{2}} - 5x^{\frac{3}{2}} + 9x^{\frac{1}{2}}$.

20. Differentiate $f(x) = \frac{3}{4}x^{12} - \frac{2}{3}x^{-8}$.

# Hard Questions

21. Using first principles, show that if $f(x) = x^{\frac{1}{3}}$ then

$$f'(x) = \frac{1}{3x^{\frac{2}{3}}}.$$

22. Explain and justify why the derivative of $f(x) = x^p$ is $f'(x) = px^{p-1}$ for any rational number $p$. In your answer, illustrate your explanation using $p = \frac{4}{3}$.

23. Differentiate $f(x) = x^{\frac{2}{3}} + x^{\frac{1}{4}}$ and simplify your answer.

24. Differentiate $f(x) = 3x^{\frac{7}{4}} - 5x^{-\frac{3}{4}} + 2$ and simplify the result, ensuring that any negative exponents are clearly presented.

25. Differentiate $f(x) = \frac{2}{5}x^{\frac{5}{4}} - \frac{1}{2}x^{\frac{1}{4}} + 8$ and express your answer in a simplified form.

26. Differentiate $f(x) = 6x^{\frac{3}{4}} - 2x^{\frac{5}{4}} + 5x^{-\frac{1}{4}}$ and factor any common factors from your final answer.

27. Given $f(x) = x^{\frac{3}{4}} - 3x^{\frac{1}{4}} + 2$, differentiate and factor your answer completely.

28. Differentiate $f(x) = \frac{8}{3}x^{\frac{7}{2}} - \frac{3}{2}x^{-\frac{5}{2}}$ and simplify the result.

29. For the function $f(x) = x^{\frac{10}{3}} - 5x^{\frac{7}{3}} + 2x^{\frac{4}{3}}$, find $f'(x)$ and express your answer in its simplest form.

30. Differentiate $f(x) = \frac{4}{3}x^6 - \frac{7}{2}x^{-3} + 5x^{\frac{1}{4}}$ and simplify your answer.