ILLAWARRA TUTORING

This worksheet is on the topic of Derivatives and indices. In this unit you will learn to differentiate functions that involve indices, expanding your ability to handle exponential expressions. You will apply the power rule to functions with integer, fractional and negative exponents. Ensure you show all your working and simplify your answers where possible.

Easy Questions

1. Differentiate $f(x) = x^3$.
2. Differentiate $f(x) = x^{\frac{1}{2}}$.
3. Differentiate $f(x) = 2x^{-1}$.
4. Differentiate $f(x) = 3x^{\frac{1}{2}}$.
5. Differentiate $f(x) = -4x^4$.

Intermediate Questions

6. Differentiate $f(x) = 5x^{\frac{7}{2}}$.
7. Differentiate $f(x) = -3x^{-3} + 2x^{\frac{1}{3}}$.
8. Differentiate $f(x) = \frac{1}{2}x^8 - 4x^{\frac{5}{2}}$.
9. Differentiate $f(x) = 7x^{\frac{2}{3}} + 9x^4 - x^{\frac{3}{2}}$.
10. Differentiate $f(x) = 3x^{\frac{2}{3}} - 5x^{-\frac{1}{3}}$.
11. Differentiate $f(x) = 6x^{\frac{3}{2}} + 2x^{\frac{1}{2}} - 8x^{\frac{5}{2}}$.
12. Differentiate $f(x) = \frac{1}{3}x^{\frac{9}{2}} - 7x^2$.
13. Differentiate $f(x) = -x^{\frac{4}{3}} + 5x^{\frac{7}{3}}$.
14. Differentiate $f(x) = 2x^{\frac{5}{2}} - 3x^{-2} + x^{\frac{1}{2}}$.
15. Differentiate $f(x) = 10x^{-\frac{1}{2}} + x^{\frac{7}{2}}$.
16. Differentiate $f(x) = x^2 + 4x^{\frac{1}{3}} - 6x^{-\frac{4}{3}}$.

17. Differentiate $f(x) = -3x^7 + 2x^{\frac{3}{2}} - x^{\frac{5}{2}}$.

18. Differentiate $f(x) = 12x^{\frac{1}{4}} - 8x^{\frac{5}{4}} + x^{\frac{9}{4}}$.

19. Differentiate $f(x) = 9x^{\frac{11}{2}} - 4x^{\frac{3}{2}} + 7x^{\frac{1}{2}}$.

20. Differentiate $f(x) = \frac{1}{2}x^{14} - \frac{3}{2}x^{-7}$.

Hard Questions

21. Using first principles, show that if $f(x) = x^{\frac{1}{2}}$ then

$$f'(x) = \frac{1}{2\sqrt{x}}.$$

22. Explain and justify why the derivative of $f(x) = x^p$ is $f'(x) = px^{p-1}$ for any rational number p. In your answer, illustrate your explanation using $p = \frac{3}{2}$.

23. Differentiate $f(x) = x^{\frac{1}{3}} + x^{\frac{1}{2}}$ and simplify your answer, writing your result in simplest index form.

24. Differentiate $f(x) = 2x^{\frac{5}{3}} - 4x^{-\frac{2}{3}} + 7$ and simplify your answer, ensuring that any negative exponents are clearly presented.

25. Differentiate $f(x) = \frac{1}{3}x^{\frac{4}{3}} - \frac{2}{5}x^{\frac{1}{3}} + 9$ and express your answer in a simplified form.

26. Differentiate $f(x) = 8x^{\frac{2}{3}} - 3x^{\frac{5}{3}} + 4x^{-\frac{1}{3}}$ and factor any common factors from your final answer.

27. Given $f(x) = x^{\frac{2}{3}} - 2x^{\frac{1}{3}} + 1$, differentiate and factor your answer completely.

28. Differentiate $f(x) = \frac{10}{3}x^{\frac{9}{2}} - \frac{5}{2}x^{-\frac{3}{2}}$ and simplify the result.

29. For the function $f(x) = x^{\frac{11}{3}} - 7x^{\frac{4}{3}} + 3x^{\frac{1}{3}}$, find $f'(x)$ and express your answer in its simplest form.

30. A function is given by $f(x) = \frac{3}{2}x^7 - \frac{5}{3}x^{-2} + 4x^{\frac{1}{3}}$. Differentiate $f(x)$ and simplify your answer.