ILLAWARRA TUTORING

This worksheet examines the properties and graphs of reciprocal functions (hyperbolas) and helps you understand how they differ from other function types. You will explore domains, asymptotes, transformations and the algebraic behaviour of these functions.

## Easy Questions

1. Determine the domain and range of the function $y = \frac{1}{x}$.
2. On a separate sheet of paper, sketch the graph of $y = \frac{1}{x}$, ensuring you label the asymptotes.
3. State the vertical asymptote of the function $y = \frac{1}{x}$.
4. Write an equation for a hyperbola of the form $y = \frac{k}{x}$ that has asymptotes $x = 0$ and $y = 0$ and passes through the point $(1, 2)$.
5. Describe how the graph of $y = \frac{1}{x}$ changes if it is shifted two units to the right.

## Intermediate Questions

6. Determine the vertical asymptote of the function $y = \frac{1}{(x-1)}$.
7. State the horizontal asymptote of the function $y = \frac{1}{(x-1)}$.
8. Solve for $x$ in the equation $\frac{1}{(x-3)} = 2$. Show all steps.
9. Given that the hyperbola $y = \frac{k}{x}$ passes through the point $(2, 4)$, find the value of $k$.
10. Write the equation of the reciprocal function obtained by shifting $y = \frac{1}{x}$ three units to the left.
11. For the function $y = \frac{1}{x}$, replace $x$ with $-x$ to find $y = f(-x)$ and determine the type of symmetry the graph has.

12. Solve $\dfrac{1}{(x+2)} = \dfrac{1}{4}$ for $x$.

13. The function $y = \dfrac{1}{x}$ is scaled vertically by a factor of 3. Write the new equation.

14. Describe the transformation that maps $f(x) = \dfrac{1}{x}$ to $g(x) = \dfrac{-2}{x+1} + 3$. List the steps in order.

15. Explain one key property that distinguishes hyperbolas (reciprocal functions) from parabolas.

16. Find the equation of a reciprocal function in the form $y = \dfrac{k}{x}$ with vertical asymptote $x = 2$ and horizontal asymptote $y = -3$. (Hint: Consider how a vertical stretch and vertical translation affect a reciprocal function.)

17. For $f(x) = \dfrac{1}{x}$, calculate $f(-0.5)$.

18. Compute the product $f(x) \times f(-x)$ for $f(x) = \dfrac{1}{x}$ and discuss the result.

19. Describe the behaviour of $y = \dfrac{1}{x}$ as $x$ tends to zero from the right.

20. Determine the $x$-coordinate(s) of the intersection between the hyperbola $y = \dfrac{1}{x}$ and the line $y = 2$, if any.

## Hard Questions

21. Prove analytically that the function $f(x) = \dfrac{1}{x}$ is symmetric about the origin.

22. The hyperbola $y = \dfrac{k}{x}$ passes through the points $(4, 3)$ and $(2, 6)$. Find the value of $k$, and explain your reasoning.

23. Find the $x$-intercepts of the function $y = \dfrac{1}{x}$ and justify your answer.

24. For $f(x) = \dfrac{1}{(x-1)} + 2$, determine the equations of both the vertical and horizontal asymptotes and explain how you obtained them.

25. Solve the equation $\dfrac{1}{(2x-1)} = \dfrac{3}{(x+2)}$. Ensure you check for any extraneous solutions.

26. Show that the function $f(x) = \dfrac{1}{x}$ is its own inverse by finding $f(f(x))$.

27. Consider the function $g(x) = \dfrac{1}{x^2}$. Explain why $g(x)$ does not represent a hyperbola.

28. Determine the equation of the reflection of the hyperbola $y = \dfrac{1}{x}$ in the line $y = x$. Provide a brief explanation.

29. Given $f(x) = \dfrac{1}{x}$ and $g(x) = \dfrac{1}{(x-2)}$, find all $x$ for which $f(x) = g(x)$ and explain your solution.

30. Derive the derivative of $f(x) = \dfrac{1}{x}$ and discuss how this derivative relates to the function's asymptotic behaviour near $x = 0$.