ILLAWARRA TUTORING

This worksheet examines the properties and graphs of reciprocal functions and explores how they differ from other function types. You will investigate asymptotes, domains, transformations and intersections for functions of the form $y = \dfrac{k}{x-h} + v$.

Easy Questions

1. Write the equation $y = \dfrac{1}{x}$ and state its vertical and horizontal asymptotes.
2. Determine the domain and range of the function $y = \dfrac{1}{x}$.
3. Evaluate $y = \dfrac{1}{x}$ at $x = 2$.
4. Using pen and paper, sketch the graph of $y = \dfrac{1}{x}$ and mark its asymptotes.
5. State whether $y = \dfrac{1}{x}$ is symmetric with respect to the y-axis, the x-axis or the origin. Provide a brief explanation.

Intermediate Questions

6. Write the equation $y = \dfrac{k}{x-h} + v$ for a reciprocal function with $k = 2$, $h = 1$ and $v = -3$. Clearly identify each parameter.
7. For $y = \dfrac{1}{x-2}$, determine the vertical and horizontal asymptotes.
8. For the function $y = \dfrac{-3}{x}$, state its asymptotes and describe the sign of y when x is positive and when x is negative.
9. Transform $y = \dfrac{1}{x}$ by shifting the graph 2 units to the right and 4 units up. Write down the new equation.
10. Determine the equation of a reciprocal function with a vertical asymptote at $x = -1$ and horizontal asymptote at $y = 2$, given that the graph passes through the point $(0, 0)$.
11. Find the intersection point of the asymptotes for the function $y = \dfrac{2}{x+3} - 1$.

12. Describe the behaviour of $y = \dfrac{1}{x}$ as x approaches 0 from the right and from the left.

13. Find the equation of a reciprocal function with a vertical asymptote $x = 4$ and a horizontal asymptote $y = -2$ that passes through $(5, 1)$.

14. Using transformations of $y = \dfrac{1}{x}$, explain how to graph $y = \dfrac{-1}{x-1} + 3$ on pen and paper, detailing each translation and reflection.

15. In a short paragraph, explain in your own words how the graph of a reciprocal function differs from that of a quadratic function.

16. Solve for x by finding the intersection of the curves $y = \dfrac{1}{x}$ and $y = x - 2$. Show all steps.

17. Solve the equation $\dfrac{1}{x-1} = \dfrac{2}{x+1}$ and state any restrictions on x.

18. Prove that the function $f(x) = \dfrac{1}{x}$ is one-to-one. Provide a brief explanation along with your proof.

19. If a reciprocal function is given by $y = \dfrac{k}{x}$ and its graph passes through the point $(2, 5)$, determine the value of k.

Hard Questions

20. Derive the equation of a reciprocal function with vertical asymptote $x = 3$ and horizontal asymptote $y = -2$ that passes through $(5, 1)$. Show all working.

21. For the function $y = \dfrac{4}{x-2}$, find the y-intercept and explain why there is no x-intercept.

22. Explain, using limit arguments, why the graph of $y = \dfrac{1}{x}$ never intersects its asymptotes.

23. Prove that $y = \dfrac{1}{x}$ is symmetric about the origin.

24. For $y = \dfrac{-2}{x+4} - 3$, determine its vertical and horizontal asymptotes and describe the effect of the constant factors and shifts on the graph compared to $y = \dfrac{1}{x}$.

25. Solve the equation $\dfrac{1}{x-2} + \dfrac{1}{x+2} = \dfrac{1}{2}$. Clearly state any restrictions on x and show all steps.

26. Demonstrate that the function $y = \dfrac{1}{x}$ is strictly decreasing on both $x > 0$ and $x < 0$. Provide a brief justification.

27. For the reciprocal function $y = \dfrac{k}{x}$, determine the values of k for which the graph lies entirely below the line $y = 0$ when $x > 0$.

28. Find the points of intersection between $y = \dfrac{3}{x-1} + 2$ and $y = \dfrac{-2}{x}$. Show all of your working.

29. For $y = \dfrac{2}{x-3} - 4$, explain in detail the sequence of transformations applied to $y = \dfrac{1}{x}$ and determine the coordinates of the intersection of its asymptotes.